SIMILARITY

How can you use proportions to turn a drawing into a mural?
APPLICATION: Scale Drawing

Murals are often created by enlarging an original drawing. Different methods are used to make sure that all parts of the enlargement are in proportion to the original drawing.

One common method used in mural making is to enlarge each piece of art by the same percentage. If a drawing is enlarged to 300% of its original size, then the length and width of the enlargement will each be three times the size of the original.

Think & Discuss

1. Describe some other common methods used to enlarge a drawing.

2. Estimate how much larger Figure 2 is than Figure 1. Can you discover a way to check your estimate?

Learn More About It

You will learn another way to enlarge a drawing in Example 4 on p. 490.

INTERNET APPLICATION LINK Visit www.mcdougallittell.com for more information about scale drawings.
What’s the chapter about?

Chapter 8 is about similar polygons. Two polygons are similar if their corresponding angles are congruent and the lengths of corresponding sides are proportional. In Chapter 8, you’ll learn

- four ways to prove triangles are similar given information about their sides and angles.
- how to use similar polygons to solve real-life problems.

### KEY VOCABULARY

**Review**
- angle bisector, p. 36
- slope, p. 165
- transformation, p. 396
- image, p. 396
- preimage, p. 396

**New**
- ratio, p. 457
- proportion, p. 459
- means, p. 459
- extremes, p. 459
- geometric mean, p. 466
- similar polygons, p. 473
- scale factor, p. 474
- dilation, p. 506
- reduction, p. 506
- enlargement, p. 506

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### SKILL REVIEW

**Find the perimeter of the figure.** (Review pp. 51–54)

1. ![Figure 1]
2. ![Figure 2]
3. ![Figure 3]

**Find the slope of the line that passes through the points.** (Review Example 2, p. 165)

4. A(0, 0) and B(4, 2)  
5. C(−1, 2) and D(6, 5)  
6. E(0, 3) and F(−4, −8)

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**Here’s a study strategy!**

Connect to the Real World

Make a list of the main topics of the chapter. Give a real-world example for each.
Ratio and Proportion

**COMPUTING RATIOS**

If \( a \) and \( b \) are two quantities that are measured in the same units, then the ratio of \( a \) to \( b \) is \( \frac{a}{b} \). The ratio of \( a \) to \( b \) can also be written as \( a:b \). Because a ratio is a quotient, its denominator cannot be zero.

Ratios are usually expressed in simplified form. For instance, the ratio of 6:8 is usually simplified as 3:4.

**EXAMPLE 1** Simplifying Ratios

Simplify the ratios.

a. \( \frac{12 \text{ cm}}{4 \text{ m}} \)

b. \( \frac{6 \text{ ft}}{18 \text{ in.}} \)

**SOLUTION**

To simplify ratios with unlike units, convert to like units so that the units divide out. Then simplify the fraction, if possible.

a. \( \frac{12 \text{ cm}}{4 \text{ m}} = \frac{12 \text{ cm}}{4 \times 100 \text{ cm}} = \frac{12}{400} = \frac{3}{100} \)

b. \( \frac{6 \text{ ft}}{18 \text{ in.}} = \frac{6 \text{ ft} \times 12 \text{ in.}}{18 \text{ in.}} = \frac{72}{18} = \frac{4}{1} \)

**ACTIVITY** Developing Concepts

**Investigating Ratios**

1. Use a tape measure to measure the circumference of the base of your thumb, the circumference of your wrist, and the circumference of your neck. Record the results in a table.

2. Compute the ratio of your wrist measurement to your thumb measurement. Then, compute the ratio of your neck measurement to your wrist measurement.

3. Compare the two ratios.

4. Compare your ratios to those of others in the class.

5. Does it matter whether you record your measurements all in inches or all in centimeters? Explain.
EXAMPLE 2 Using Ratios

The perimeter of rectangle $ABCD$ is 60 centimeters. The ratio of $AB : BC$ is $3:2$. Find the length and width of the rectangle.

**SOLUTION**

Because the ratio of $AB : BC$ is $3:2$, you can represent the length $AB$ as $3x$ and the width $BC$ as $2x$.

\[ 2l + 2w = P \quad \text{Formula for perimeter of rectangle} \]
\[ 2(3x) + 2(2x) = 60 \quad \text{Substitute for } l, w, \text{ and } P. \]
\[ 6x + 4x = 60 \quad \text{Multiply.} \]
\[ 10x = 60 \quad \text{Combine like terms.} \]
\[ x = 6 \quad \text{Divide each side by 10.} \]

So, $ABCD$ has a length of 18 centimeters and a width of 12 centimeters.

EXAMPLE 3 Using Extended Ratios

The measure of the angles in $\triangle JKL$ are in the extended ratio of $1:2:3$. Find the measures of the angles.

**SOLUTION**

Begin by sketching a triangle. Then use the extended ratio of $1:2:3$ to label the measures of the angles as $x^\circ$, $2x^\circ$, and $3x^\circ$.

\[ x^\circ + 2x^\circ + 3x^\circ = 180^\circ \quad \text{Triangle Sum Theorem} \]
\[ 6x = 180 \quad \text{Combine like terms.} \]
\[ x = 30 \quad \text{Divide each side by 6.} \]

So, the angle measures are $30^\circ$, $2(30^\circ) = 60^\circ$, and $3(30^\circ) = 90^\circ$.

EXAMPLE 4 Using Ratios

The ratios of the side lengths of $\triangle DEF$ to the corresponding side lengths of $\triangle ABC$ are $2:1$. Find the unknown lengths.

**SOLUTION**

- $DE$ is twice $AB$ and $DE = 8$, so $AB = \frac{1}{2}(8) = 4$.
- Using the Pythagorean Theorem, you can determine that $BC = 5$.
- $DF$ is twice $AC$ and $AC = 3$, so $DF = 2(3) = 6$.
- $EF$ is twice $BC$ and $BC = 5$, so $EF = 2(5) = 10$. 
GOAL 2 USING PROPORTIONS

An equation that equates two ratios is a proportion. For instance, if the ratio \( \frac{a}{b} \) is equal to the ratio \( \frac{c}{d} \), then the following proportion can be written:

\[
\frac{a}{b} = \frac{c}{d}
\]

The numbers \( a \) and \( d \) are the \textbf{extremes} of the proportion. The numbers \( b \) and \( c \) are the \textbf{means} of the proportion.

**PROPERTIES OF PROPORTIONS**

1. **CROSS PRODUCT PROPERTY** The product of the extremes equals the product of the means.
   \[
   \text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = bc.
   \]

2. **RECIPROCAL PROPERTY** If two ratios are equal, then their reciprocals are also equal.
   \[
   \text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{b}{a} = \frac{d}{c}.
   \]

To solve the proportion you find the value of the variable.

**EXAMPLE 5 Solving Proportions**

Solve the proportions.

\( \frac{4}{x} = \frac{5}{7} \)

\( \frac{3}{y + 2} = \frac{2}{y} \)

**SOLUTION**

\( \frac{4}{x} = \frac{5}{7} \)

Write original proportion.

Write reciprocal property

Multiply each side by \( 4 \).

Simplify.

\( \frac{3}{y + 2} = \frac{2}{y} \)

Write original proportion.

Cross product property

Distributive property

Subtract \( 2y \) from each side.

The solution is 4. Check this by substituting in the original proportion.
EXAMPLE 6 Solving a Proportion

PAINTING The photo shows Bev Dolittle’s painting *Music in the Wind*. Her actual painting is 12 inches high. How wide is it?

SOLUTION
You can reason that in the photograph all measurements of the artist’s painting have been reduced by the same ratio. That is, the ratio of the actual width to the reduced width is equal to the ratio of the actual height to the reduced height.

The photograph is $1\frac{1}{4}$ inches by $4\frac{3}{8}$ inches.

So, the actual painting is 42 inches wide.

EXAMPLE 7 Solving a Proportion

Estimate the length of the hidden flute in Bev Doolittle’s actual painting.

SOLUTION
In the photo, the flute is about $1\frac{7}{8}$ inches long. Using the reasoning from above you can say that:

$$\frac{\text{Length of flute in painting}}{\text{Length of flute in photo}} = \frac{\text{Height of painting}}{\text{Height of photo}}$$

$$\frac{f}{1.875} = \frac{12}{1.25} \quad \text{Substitute.}$$

$$f = 18 \quad \text{Multiply each side by 1.875 and simplify.}$$

So, the flute is about 18 inches long in the painting.
**GUIDED PRACTICE**

1. In the proportion \( \frac{x}{s} = \frac{p}{q} \), the variables \( s \) and \( p \) are the _?_ of the proportion and \( r \) and \( q \) are the _?_ of the proportion.

**ERROR ANALYSIS** In Exercises 2 and 3, find and correct the errors.

2. A table is 18 inches wide and 3 feet long. The ratio of length to width is 1 : 6.

3. \( \frac{10}{x + 6} = \frac{4}{x} \)
   
   \[
   10x = 4(x + 6) \\
   6x = 6 \\
   x = 1
   \]

Skill Check ✓

Given that the track team won 8 meets and lost 2, find the ratios.

4. What is the ratio of wins to losses? What is the ratio of losses to wins?

5. What is the ratio of wins to the total number of track meets?

In Exercises 6–8, solve the proportion.

6. \( \frac{2}{x} = \frac{3}{9} \)

7. \( \frac{5}{8} = \frac{6}{z} \)

8. \( \frac{2}{b + 3} = \frac{4}{b} \)

9. The ratio \( BC: DC \) is 2 : 9. Find the value of \( x \).

**PRACTICE AND APPLICATIONS**

- **Simplifying Ratios** Simplify the ratio.
  
  10. \( \frac{16}{24} \) students
  
  11. \( \frac{48}{8} \) marbles
  
  12. \( \frac{22}{52} \) feet
  
  13. \( \frac{6}{9} \) meters

- **Writing Ratios** Find the width to length ratio of each rectangle. Then simplify the ratio.
  
  14. \( \frac{20 \text{ mm}}{16 \text{ mm}} \)
  
  15. \( \frac{10 \text{ cm}}{7.5 \text{ cm}} \)
  
  16. \( \frac{20 \text{ ft}}{12 \text{ in.}} \)

- **Converting Units** Rewrite the fraction so that the numerator and denominator have the same units. Then simplify.
  
  17. \( \frac{3 \text{ ft}}{12 \text{ in.}} \)
  
  18. \( \frac{60 \text{ cm}}{1 \text{ m}} \)
  
  19. \( \frac{350 \text{ g}}{1 \text{ kg}} \)
  
  20. \( \frac{2 \text{ mi}}{3000 \text{ ft}} \)
  
  21. \( \frac{6 \text{ yd}}{10 \text{ ft}} \)
  
  22. \( \frac{2 \text{ lb}}{20 \text{ oz}} \)
  
  23. \( \frac{400 \text{ m}}{0.5 \text{ km}} \)
  
  24. \( \frac{20 \text{ oz}}{4 \text{ lb}} \)
**Finding Ratios** Use the number line to find the ratio of the distances.

\[ \frac{AB}{CD} = \? \]
\[ \frac{BD}{CF} = \? \]
\[ \frac{BF}{AD} = \? \]
\[ \frac{CF}{AB} = \? \]

25. The perimeter of a rectangle is 84 feet. The ratio of the width to the length is 2:5. Find the length and the width.

26. The area of a rectangle is 108 cm². The ratio of the width to the length is 3:4. Find the length and the width.

27. The measures of the angles in a triangle are in the extended ratio of 1:4:7. Find the measures of the angles.

28. The measures of the angles in a triangle are in the extended ratio of 2:15:19. Find the measures of the angles.

**Solving Proportions** Solve the proportion.

29. \[ \frac{x}{4} = \frac{5}{7} \]

30. \[ \frac{4}{b} = \frac{10}{3} \]

31. \[ \frac{5}{x + 3} = \frac{4}{x} \]

32. \[ \frac{3x - 8}{6} = \frac{2x}{10} \]

33. \[ \frac{y}{8} = \frac{9}{10} \]

34. \[ \frac{30}{5} = \frac{14}{c} \]

35. \[ \frac{7}{z} = \frac{10}{25} \]

36. \[ \frac{16}{3} = \frac{d}{6} \]

37. \[ \frac{4}{y - 3} = \frac{8}{y} \]

38. \[ \frac{7}{2z + 5} = \frac{3}{z} \]

39. \[ \frac{5y - 8}{7} = \frac{5y}{6} \]

40. \[ \frac{4}{2z + 6} = \frac{10}{7z - 2} \]

**Using Proportions** In Exercises 45–47, the ratio of the width to the length for each rectangle is given. Solve for the variable.

41. \[ AB:BC \] is 3:8.

42. \[ EF:FG \] is 4:5.

43. \[ JK:KL \] is 2:3.

44. \[ \frac{2x}{10} \]

45. \[ \frac{2y}{7} \]

46. \[ \frac{8}{y} \]

47. \[ \frac{5y}{6} \]

**Science Connection** Use the following information.

The table gives the ratios of the gravity of four different planets to the gravity of Earth. Round your answers to the nearest whole number.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Venus</th>
<th>Mars</th>
<th>Jupiter</th>
<th>Pluto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of gravity</td>
<td>9/10</td>
<td>38/100</td>
<td>236/100</td>
<td>7/100</td>
</tr>
</tbody>
</table>

48. Which of the planets listed above has a gravity closest to the gravity of Earth?

49. Estimate how much a person who weighs 140 pounds on Earth would weigh on Venus, Mars, Jupiter, and Pluto.

50. If a person weighed 46 pounds on Mars, estimate how much he or she would weigh on Earth.
**Baseball Bat Sculpture** A huge, free-standing baseball bat sculpture stands outside a sports museum in Louisville, Kentucky. It was patterned after Babe Ruth’s 35 inch bat. The sculpture is 120 feet long. Round your answers to the nearest tenth of an inch.

51. How long is the sculpture in inches?
52. The diameter of the sculpture near the base is 9 feet. Estimate the corresponding diameter of Babe Ruth’s bat.
53. The diameter of the handle of the sculpture is 3.5 feet. Estimate the diameter of the handle of Babe Ruth’s bat.

**Using Proportions** In Exercises 54–56, the ratio of two side lengths of the triangle is given. Solve for the variable.

54. \( \frac{PQ}{QR} = \frac{3}{4} \)
55. \( \frac{SU}{ST} = \frac{4}{1} \)
56. \( \frac{WX}{XV} = \frac{5}{7} \)

**Pythagorean Theorem** The ratios of the side lengths of \( \triangle PQR \) to the corresponding side lengths of \( \triangle STU \) are 1:3. Find the unknown lengths.

57.

58.

**Gulliver’s Travels** In Exercises 59–61, use the following information.

Gulliver’s Travels was written by Jonathan Swift in 1726. In the story, Gulliver is shipwrecked and wanders ashore to the island of Lilliput. The average height of the people in Lilliput is 6 inches.

59. Gulliver is 6 feet tall. What is the ratio of his height to the average height of a Lilliputian?

60. After leaving Lilliput, Gulliver visits the island of Brobdingnag. The ratio of the average height of these natives to Gulliver’s height is proportional to the ratio of Gulliver’s height to the average height of a Lilliputian. What is the average height of a Brobdingnagian?

61. What is the ratio of the average height of a Brobdingnagian to the average height of a Lilliputian?
**USING ALGEBRA** You are given an extended ratio that compares the lengths of the sides of the triangle. Find the lengths of all unknown sides.

64. $GH:HR:GR$ is $5:5:6$.

**Test Preparation**

65. **MULTIPLE CHOICE** For planting roses, a gardener uses a special mixture of soil that contains sand, peat moss, and compost in the ratio $2:5:3$. How many pounds of compost does she need to add if she uses three 10 pound bags of peat moss?

- A 12  
- B 14  
- C 15  
- D 18  
- E 20

66. **MULTIPLE CHOICE** If the measures of the angles of a triangle have the ratio $2:3:7$, the triangle is

- A acute.  
- B right.  
- C isosceles.  
- D obtuse.  
- E equilateral.

**Challenge**

67. **FINDING SEGMENT LENGTHS** Suppose the points $B$ and $C$ lie on $AD$. What is the length of $AC$ if $\frac{AB}{BD} = \frac{2}{3}$, $\frac{CD}{AC} = \frac{1}{9}$, and $BD = 24$?

**MIXED REVIEW**

**FINDING UNKNOWN MEASURES** Use the figure shown, in which $\triangle STU \cong \triangle XWV$. (Review 4.2)

68. What is the measure of $\angle X$?  
69. What is the measure of $\angle V$?  
70. What is the measure of $\angle T$?  
71. What is the measure of $\angle U$?  
72. Which side is congruent to $TU$?

**FINDING COORDINATES** Find the coordinates of the endpoints of each midsegment shown in red. (Review 5.4 for 8.2)

73.  
74.  
75.  

76. A line segment has endpoints $A(1, -3)$ and $B(6, -7)$. Graph $AB$ and its image $A'B'$ if $AB$ is reflected in the line $x = 2$. (Review 7.2)
Problem Solving in Geometry with Proportions

GOAL 1 USING PROPERTIES OF PROPORTIONS

In Lesson 8.1, you studied the reciprocal property and the cross product property. Two more properties of proportions, which are especially useful in geometry, are given below.

You can use the cross product property and the reciprocal property to help prove these properties in Exercises 36 and 37.

ADDITIONAL PROPERTIES OF PROPORTIONS

3. If \( \frac{a}{b} = \frac{c}{d} \), then \( \frac{a}{c} = \frac{b}{d} \).

4. If \( \frac{a}{b} = \frac{c}{d} \), then \( \frac{a + b}{b} = \frac{c + d}{d} \).

EXAMPLE 1 Using Properties of Proportions

Tell whether the statement is true.

a. If \( \frac{p}{6} = \frac{r}{10} \), then \( \frac{p}{r} = \frac{3}{5} \).

b. If \( \frac{a}{3} = \frac{c}{4} \), then \( \frac{a + 3}{3} = \frac{c + 4}{4} \).

SOLUTION

a. \( \frac{p}{6} = \frac{r}{10} \) Given

\( \frac{p}{r} = \frac{6}{10} \) If \( \frac{a}{b} = \frac{c}{d} \), then \( \frac{a}{c} = \frac{b}{d} \).

\( \frac{p}{r} = \frac{3}{5} \) Simplify.

The statement is true.

b. \( \frac{a}{3} = \frac{c}{4} \) Given

\( \frac{a + 3}{3} = \frac{c + 4}{4} \) If \( \frac{a}{b} = \frac{c}{d} \), then \( \frac{a + b}{b} = \frac{c + d}{d} \).

Because \( \frac{c + 4}{4} \neq \frac{c + 3}{4} \), the conclusions are not equivalent.

The statement is false.
**EXAMPLE 2** Using Properties of Proportions

In the diagram \( \frac{AB}{BD} = \frac{AC}{CE} \). Find the length of \( BD \).

**SOLUTION**

\[
\frac{AB}{BD} = \frac{AC}{CE} \quad \text{Given}
\]

\[
\frac{16}{x} = \frac{30 - 10}{10} \quad \text{Substitute.}
\]

\[
\frac{16}{x} = \frac{20}{10} \quad \text{Simplify.}
\]

\[
20x = 160 \quad \text{Cross product property}
\]

\[
x = 8 \quad \text{Divide each side by 20.}
\]

So, the length of \( BD \) is 8.

The geometric mean of two positive numbers \( a \) and \( b \) is the positive number \( x \) such that \( \frac{a}{x} = \frac{x}{b} \). If you solve this proportion for \( x \), you find that \( x = \sqrt{a \cdot b} \), which is a positive number.

For example, the geometric mean of 8 and 18 is 12, because \( \frac{8}{12} = \frac{12}{18} \), and also because \( \sqrt{8 \cdot 18} = \sqrt{144} = 12 \).

**EXAMPLE 3** Using a Geometric Mean

**PAPER SIZES** International standard paper sizes are commonly used all over the world. The various sizes all have the same width-to-length ratios. Two sizes of paper are shown, called A4 and A3. The distance labeled \( x \) is the geometric mean of 210 mm and 420 mm. Find the value of \( x \).

**SOLUTION**

\[
\frac{210}{x} = \frac{x}{420} \quad \text{Write proportion.}
\]

\[
x^2 = 210 \cdot 420 \quad \text{Cross product property}
\]

\[
x = \sqrt{210 \cdot 420} \quad \text{Simplify.}
\]

\[
x = \sqrt{210 \cdot 210 \cdot 2} \quad \text{Factor.}
\]

\[
x = 210\sqrt{2} \quad \text{Simplify.}
\]

So, the geometric mean of 210 and 420 is \( 210\sqrt{2} \), or about 297. So, the distance labeled \( x \) in the diagram is about 297 mm.
**GOAL 2 USING PROPORTIONS IN REAL LIFE**

In general, when solving word problems that involve proportions, there is more than one correct way to set up the proportion.

**EXAMPLE 4 Solving a Proportion**

**MODEL BUILDING** A scale model of the Titanic is 107.5 inches long and 11.25 inches wide. The Titanic itself was 882.75 feet long. How wide was it?

**SOLUTION**

One way to solve this problem is to set up a proportion that compares the measurements of the Titanic to the measurements of the scale model.

\[
\frac{\text{Width of Titanic}}{\text{Width of model ship}} = \frac{\text{Length of Titanic}}{\text{Length of model ship}}
\]

\[
\text{Width of Titanic} = x \text{ (feet)}
\]

\[
\text{Width of model ship} = 11.25 \text{ (inches)}
\]

\[
\text{Length of Titanic} = 882.75 \text{ (feet)}
\]

\[
\text{Length of model ship} = 107.5 \text{ (inches)}
\]

\[
\frac{x \text{ ft}}{11.25 \text{ in.}} = \frac{882.75 \text{ ft}}{107.5 \text{ in.}}
\]

\[
x = \frac{11.25 \cdot (882.75)}{107.5}
\]

\[
x \approx 92.4
\]

So, the Titanic was about 92.4 feet wide.

Notice that the proportion in Example 4 contains measurements that are not in the same units. When writing a proportion with unlike units, the numerators should have the same units and the denominators should have the same units.
**GUIDED PRACTICE**

**Vocabulary Check ✓**

1. If \( x \) is the geometric mean of two positive numbers \( a \) and \( b \), write a proportion that relates \( a \), \( b \), and \( x \). \( \frac{a}{x} = \frac{x}{b} \)

**Concept Check ✓**

2. If \( \frac{x}{4} = \frac{y}{5} \), then \( \frac{x + 4}{4} = \frac{y + 5}{5} \)

3. If \( \frac{b}{6} = \frac{c}{2} \), then \( \frac{b}{c} = \frac{6}{2} \); or \( 3 \)

**Skill Check ✓**

4. Decide whether the statement is true or false. true

   \[ \frac{r}{s} = \frac{6}{15}, \text{ then } \frac{15}{s} = \frac{6}{r} \]

5. Find the geometric mean of 3 and 12. \( 6 \)

6. In the diagram \( \frac{AB}{BC} = \frac{AD}{DE} \).
   Substitute the known values into the proportion and solve for \( DE \). \( 9 \)

7. **United States Flag** The official height-to-width ratio of the United States flag is 1:1.9. If a United States flag is 6 feet high, how wide is it? \( 11.4 \text{ ft} \)

8. **United States Flag** The blue portion of the United States flag is called the union. What is the ratio of the height of the union to the height of the flag? \( \frac{7}{13} \)

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**PRACTICE AND APPLICATIONS**

**LOGICAL REASONING** Complete the sentence.

9. If \( \frac{2}{x} = \frac{7}{y} \), then \( \frac{2}{y} = \frac{7}{x} \); \( \frac{x}{y} \)

10. If \( \frac{x}{6} = \frac{y}{34} \), then \( \frac{x}{y} = \frac{6}{34} \); or \( 3 \)

11. If \( \frac{x}{5} = \frac{y}{12} \), then \( \frac{x + 5}{y} = \frac{12}{x} \); \( \frac{x + y}{12} \)

12. If \( \frac{13}{7} = \frac{x}{y} \), then \( \frac{20}{7} = \frac{x + y}{y} \); \( \frac{x + y}{y} \)

**LOGICAL REASONING** Decide whether the statement is true or false.

13. If \( \frac{7}{a} = \frac{b}{2} \), then \( \frac{7 + a}{a} = \frac{b + 2}{2} \); true

14. If \( \frac{3}{4} = \frac{p}{r} \), then \( \frac{4}{3} = \frac{p}{r} \); false

15. If \( \frac{c}{6} = \frac{d + 2}{10} \), then \( \frac{c}{d + 2} = \frac{6}{10} \); true

16. If \( \frac{12 + m}{12} = \frac{3 + n}{n} \), then \( \frac{m}{12} = \frac{3}{n} \); true

**GEOMETRIC MEAN** Find the geometric mean of the two numbers.

17. 3 and 27 \( 9 \)

18. 4 and 16 \( 8 \)

19. 7 and 28 \( 14 \)

20. 2 and 40 \( 4\sqrt{5} \)

21. 8 and 20 \( 4\sqrt{10} \)

22. 5 and 15 \( 5\sqrt{3} \)
**Properties of Proportions** Use the diagram and the given information to find the unknown length.

23. **GIVEN** \( \frac{AB}{BD} = \frac{AC}{CE} \), find \( BD \).  
24. **GIVEN** \( \frac{VW}{WF} = \frac{VX}{XZ} \), find \( VX \).

25. **GIVEN** \( \frac{BT}{TR} = \frac{ES}{SL} \), find \( TR \).
26. **GIVEN** \( \frac{SP}{SK} = \frac{SO}{SJ} \), find \( SQ \).

27. **GIVEN** \( \frac{LJ}{JN} = \frac{MK}{KP} \), find \( JN \).
28. **GIVEN** \( \frac{OU}{OS} = \frac{RV}{RT} \), find \( SU \).

**Blueprints** In Exercises 29 and 30, use the blueprint of the house in which \( \frac{1}{16} \) inch = 1 foot. Use a ruler to approximate the dimension.

29. Find the approximate width of the house to the nearest 5 feet.  
   about 25 ft
30. Find the approximate length of the house to the nearest 5 feet.  
   about 40 ft

31. **Batting Average** The batting average of a baseball player is the ratio of the number of hits to the number of official at-bats. In 1998, Sammy Sosa of the Chicago Cubs had 643 official at-bats and a batting average of .308. Use the following verbal model to find the number of hits Sammy Sosa got.

\[
\text{Number of hits} = \text{Batting average} \times \text{Number of at-bats}
\]

32. **Currency Exchange** Natalie has relatives in Russia. She decides to take a trip to Russia to visit them. She took 500 U.S. dollars to the bank to exchange for Russian rubles. The exchange rate on that day was 22.76 rubles per U.S. dollar. How many rubles did she get in exchange for the 500 U.S. dollars?  
   Source: Russia Today 11,380 rubles
35. Each side of the equation represents the slope of the line through two of the points; if the points are collinear, the slopes are the same.

40. Sample answer: Construct a ramp consisting of two ramps in opposite directions, each 18 ft long. The first should be 3 ft high at its beginning and \( 1 \frac{1}{2} \) ft high at its end, for a rise:run ratio of \( \frac{1}{12} \). The second would be \( 1 \frac{1}{2} \) ft high at its beginning and ground level at its end. The second ramp also has a rise:run ratio of \( \frac{1}{12} \).

43. If the two sizes share a dimension, the shorter dimension of A5 paper must be the longer dimension of A6 paper. That is, the length of A6 paper must be 148 mm. Let \( x \) be the width of A6 paper; 148 is the geometric mean of \( x \) and 210. Then \( \frac{x}{148} = \frac{210}{x} \), and \( x \approx 104 \) mm.

33. **COORDINATE GEOMETRY** The points \((-4, -1), (1, 1), \) and \((x, 5)\) are collinear. Find the value of \( x \) by solving the proportion below.  
\[
\frac{1 - (-1)}{1 - (-4)} = \frac{5 - 1}{x - 1}
\]

34. **COORDINATE GEOMETRY** The points \((2, 8), (6, 18), \) and \((8, y)\) are collinear. Find the value of \( y \) by solving the proportion below.  
\[
\frac{18 - 8}{6 - 2} = \frac{y - 18}{8 - 6}
\]

35. **CRITICAL THINKING** Explain why the method used in Exercises 33 and 34 is a correct way to express that three given points are collinear. See margin.

36. **PROOF** Prove property 3 of proportions (see page 465). See margin. 
If \( \frac{a}{b} = \frac{c}{d} \), then \( \frac{a}{c} = \frac{b}{d} \).

37. **PROOF** Prove property 4 of proportions (see page 465). See margin. 
If \( \frac{a}{b} = \frac{c}{d} \), then \( \frac{a}{b} + \frac{c}{d} = \frac{c + d}{d} \).

**RAMP DESIGN** Assume that a wheelchair ramp has a slope of \( \frac{1}{12} \), which is the maximum slope recommended for a wheelchair ramp.

38. A wheelchair ramp has a 15 foot run. What is its rise?  \( 1 \frac{1}{4} \) ft

39. A wheelchair ramp rises 2 feet. What is its run? 24 ft

40. You are constructing a wheelchair ramp that must rise 3 feet. Because of space limitations, you cannot build a continuous ramp with a length greater than 21 feet. Design a ramp that solves this problem. See margin.

**FOCUS ON PEOPLE** Part of the Lewis and Clark Trail on which Sacagawea acted as guide is now known as the Lolo Trail. The map, which shows a portion of the trail, has a scale of 1 inch = 6.7 miles.

41. Use a ruler to estimate the distance (measured in a straight line) between Lewis and Clark Grove and Pheasant Camp. Then calculate the actual distance in miles. About \( \frac{3}{8} \) in.; about \( 2 \frac{1}{2} \) mi

42. Estimate the distance along the trail between Portable Soup Camp and Full Stomach Camp. Then calculate the actual distance in miles. About \( 1 \frac{1}{4} \) in.; about \( 8 \frac{3}{8} \) mi

43. **Writing** Size A5 paper has a width of 148 mm and a length of 210 mm. Size A6, which is the next smaller size, shares a dimension with size A5. Use the proportional relationship stated in Example 3 and geometric mean to explain how to determine the length and width of size A6 paper. See margin.
44. **MULTIPLE CHOICE** There are 24 fish in an aquarium. If \( \frac{1}{8} \) of the fish are tetras, and \( \frac{2}{3} \) of the remaining fish are guppies, how many guppies are in the aquarium? 

- A 2
- B 3
- C 10
- D 14
- E 16

45. **MULTIPLE CHOICE** A basketball team had a ratio of wins to losses of 3:1. After winning 6 games in a row, the team’s ratio of wins to losses was 5:1. How many games had the team won before it won the 6 games in a row?  

- A 3
- B 6
- C 9
- D 15
- E 24

46. **GOLDEN RECTANGLE** A golden rectangle has its length and width in the golden ratio. If you cut a square away from a golden rectangle, the shape that remains is also a golden rectangle.

\[ \frac{1 + \sqrt{5}}{2} \]

a. The diagram indicates that \( 1 + \sqrt{5} = 2 + x \). Find \( x \).

b. To prove that the large and small rectangles are both golden rectangles, show that \( \frac{1 + \sqrt{5}}{2} = \frac{2}{x} \).

c. Give a decimal approximation for the golden ratio to six decimal places.

1.618034

47. **FINDING AREA** Find the area of the figure described. (Review 1.7)

48. Square: side = 3 cm  
9 cm²

49. Triangle: base = 13 cm, height = 4 cm  
26 cm²

50. Circle: diameter = 11 ft  
about 95 ft²

51. **FINDING ANGLE MEASURES** Find the angle measures. (Review 6.5 for 8.3)

52. \( \angle A = 109°, \angle C = 52° \)

53. \( \angle A = 109°, \angle C = 52° \)

54. \( \angle A = 115°, \angle A = \angle D = 65° \)

55. \( \angle A = \angle B = 110°, \angle C = 67° \)

56. \( \angle A = \angle B = 110°, \angle C = 67° \)

57. **PENTAGON** Describe any symmetry in a regular pentagon \( ABCDE \). (Review 7.2, 7.3)
8.3 Similar Polygons

IDENTIFYING SIMILAR POLYGONS

When there is a correspondence between two polygons such that their corresponding angles are congruent and the lengths of corresponding sides are proportional the two polygons are called similar polygons.

In the diagram, \(ABCD\) is similar to \(EFGH\). The symbol \(\sim\) is used to indicate similarity. So, \(ABCD \sim EFGH\).

Writing Similarity Statements

Pentagons \(JKLMN\) and \(STUVW\) are similar. List all the pairs of congruent angles. Write the ratios of the corresponding sides in a statement of proportionality.

**Solution**

Because \(JKLMN \sim STUVW\), you can write \(\angle J \cong \angle S\), \(\angle K \cong \angle T\), \(\angle L \cong \angle U\), \(\angle M \cong \angle V\), and \(\angle N \cong \angle W\).

You can write the statement of proportionality as follows:

\[
\frac{JK}{ST} = \frac{KL}{TU} = \frac{LM}{UV} = \frac{MN}{VW} = \frac{NJ}{WS}.
\]

Comparing Similar Polygons

Decide whether the figures are similar. If they are similar, write a similarity statement.

**Solution**

As shown, the corresponding angles of \(WXYZ\) and \(PQRS\) are congruent. Also, the corresponding side lengths are proportional.

\[
\frac{WX}{PQ} = \frac{15}{10} = \frac{3}{2}, \quad \frac{XY}{QR} = \frac{6}{4} = \frac{3}{2}, \quad \frac{YZ}{RS} = \frac{9}{6} = \frac{3}{2}, \quad \frac{ZW}{SP} = \frac{12}{8} = \frac{3}{2}.
\]

So, the two figures are similar and you can write \(WXYZ \sim PQRS\).
**GOAL 2 Using Similar Polygons in Real Life**

**Example 3 Comparing Photographic Enlargements**

**Poster Design** You have been asked to create a poster to advertise a field trip to see the Liberty Bell. You have a 3.5 inch by 5 inch photo that you want to enlarge. You want the enlargement to be 16 inches wide. How long will it be?

**Solution** To find the length of the enlargement, you can compare the enlargement to the original measurements of the photo.

\[
\frac{16 \text{ in.}}{3.5 \text{ in.}} = \frac{x \text{ in.}}{5 \text{ in.}}
\]

\[
x = \frac{16}{3.5} \cdot 5
\]

\[
x \approx 22.9 \text{ inches}
\]

The length of the enlargement will be about 23 inches.

If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**. In Example 2 on the previous page, the common ratio of \( \frac{3}{2} \) is the scale factor of \( WXYZ \) to \( PQRS \).

**Example 4 Using Similar Polygons**

The rectangular patio around a pool is similar to the pool as shown. Calculate the scale factor of the patio to the pool, and find the ratio of their perimeters.

**Solution** Because the rectangles are similar, the scale factor of the patio to the pool is 48 ft : 32 ft, which is \( \frac{3}{2} \) in simplified form.

The perimeter of the patio is \( 2(24) + 2(48) = 144 \) feet and the perimeter of the pool is \( 2(16) + 2(32) = 96 \) feet. The ratio of the perimeters is \( \frac{144}{96} \) or \( \frac{3}{2} \).

Notice in Example 4 that the ratio of the perimeters is the same as the scale factor of the rectangles. This observation is generalized in the following theorem. You are asked to prove Theorem 8.1 for two similar rectangles in Exercise 45.
**Example 5** Using Similar Polygons

Quadrilateral $JKLM$ is similar to quadrilateral $PQRS$.

Find the value of $z$.

**Solution**

Set up a proportion that contains $PQ$.

$$\frac{KL}{QR} = \frac{JK}{PQ}$$

Write proportion.

$$\frac{15}{6} = \frac{10}{z}$$

Substitute.

$$z = 4$$

Cross multiply and divide by 15.

---

**Guided Practice**

1. If two polygons are similar, must they also be congruent? Explain.

2. Decide whether the figures are similar. Explain your reasoning.

3. In the diagram, $TUVW \sim ABCD$.

4. List all pairs of congruent angles and write the statement of proportionality for the polygons.

5. Find the scale factor of $TUVW$ to $ABCD$.

6. Find the length of $TW$.

7. Find the measure of $\angle TUV$. 

**Writing Similarity Statements** Use the information given to list all pairs of congruent angles and write the statement of proportionality for the figures.

8. \( \triangle DEF \sim \triangle PQR \)
9. \( \square JKLM \sim \square WXYZ \)
10. \( \square QRSTU \sim \square ABCDE \)

**Determining Similarity** Decide whether the quadrilaterals are similar. Explain your reasoning.

11. \( \square ABCD \) and \( \square FGHE \)
12. \( \square ABCD \) and \( \square JKLM \)
13. \( \square ABCD \) and \( \square PQRS \)
14. \( \square JKLM \) and \( \square PQRS \)

**Determining Similarity** Decide whether the polygons are similar. If so, write a similarity statement.

15.

16.

17.

18.

**Using Similar Polygons** \( \triangle PQRS \sim \triangle JKLM \).

19. Find the scale factor of \( \triangle PQRS \) to \( \triangle JKLM \).
20. Find the scale factor of \( \triangle JKLM \) to \( \triangle PQRS \).
21. Find the values of \( w \), \( x \), and \( y \).
22. Find the perimeter of each polygon.
23. Find the ratio of the perimeter of \( \triangle PQRS \) to the perimeter of \( \triangle JKLM \).
**Using Similar Polygons** \( \square ABCD \sim \square EFGH. \)

24. Find the scale factor of \( \square ABCD \) to \( \square EFGH \).

25. Find the length of \( EH \).

26. Find the measure of \( \angle G \).

27. Find the perimeter of \( \square EFGH \).

28. Find the ratio of the perimeter of \( \square EFGH \) to the perimeter of \( \square ABCD \).

**Determining Similarity** Decide whether the polygons are similar. If so, find the scale factor of Figure A to Figure B.

29. 30.

**Logical Reasoning** Tell whether the polygons are always, sometimes, or never similar.

31. Two isosceles triangles 
32. Two regular polygons 
33. Two isosceles trapezoids 
34. Two rhombuses 
35. Two squares 
36. An isosceles and a scalene triangle 
37. Two equilateral triangles 
38. A right and an isosceles triangle

**Using Algebra** The two polygons are similar. Find the values of \( x \) and \( y \).

39. 40.

**TV Screens** In Exercises 43 and 44, use the following information.

Television screen sizes are based on the length of the diagonal of the screen. The aspect ratio refers to the length to width ratio of the screen. A standard 27 inch analog television screen has an aspect ratio of 4:3. A 27 inch digital television screen has an aspect ratio of 16:9.

43. Make a scale drawing of each television screen. Use proportions and the Pythagorean Theorem to calculate the lengths and widths of the screens in inches.

44. Are the television screens similar? Explain.
45. **Proof** Prove Theorem 8.1 for two similar rectangles.

**Given** \( \triangle ABCD \sim \triangle EFGH \)

**Prove** \( \frac{\text{perimeter of } ABCD}{\text{perimeter of } EFGH} = \frac{AB}{EF} \)

46. **Scale** The ratio of the perimeter of \( WXYZ \) to the perimeter of \( QRST \) is 7.5:2. Find the scale factor of \( QRST \) to \( WXYZ \).

47. **Scale** The ratio of one side of \( \triangle CDE \) to the corresponding side of similar \( \triangle FGH \) is 2:5. The perimeter of \( \triangle FGH \) is 28 inches. Find the perimeter of \( \triangle CDE \).

48. **Scale** The perimeter of \( \square PQRS \) is 94 centimeters. The perimeter of \( \square JKLM \) is 18.8 centimeters, and \( \square JKLM \sim \square PQRS \). The lengths of the sides of \( \square PQRS \) are 15 centimeters and 32 centimeters. Find the scale factor of \( \square PQRS \) to \( \square JKLM \), and the lengths of the sides of \( \square JKLM \).

49. **Multi-Step Problem** Use the similar figures shown. The scale factor of Figure 1 to Figure 2 is 7:10.

   a. Copy and complete the table.

   |

<table>
<thead>
<tr>
<th>Figure 1</th>
<th>Figure 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>BC</td>
</tr>
<tr>
<td>6.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

   b. Graph the data in the table. Let \( x \) represent the length of a side in Figure 1 and let \( y \) represent the length of the corresponding side in Figure 2. Determine an equation that relates \( x \) and \( y \).

   c. **Analyzing Data** The equation you obtained in part (b) should be linear. What is its slope? How does its slope compare to the scale factor?

   **Challenge** Use the following information in Exercises 50–52.

   From your perspective on Earth during a total eclipse of the sun, the moon is directly in line with the sun and blocks the sun’s rays. The ratio of the radius of the moon to its distance to Earth is about the same as the ratio of the radius of the sun to its distance to Earth.

   Distance between Earth and the moon: 240,000 miles
   Distance between Earth and the sun: 93,000,000 miles
   Radius of the sun: 432,500 miles

   50. Make a sketch of Earth, the moon, and the sun during a total eclipse of the sun. Include the given distances in your sketch.

   51. Your sketch should contain some similar triangles. Use the similar triangles in your sketch to explain a total eclipse of the sun.

   52. Write a statement of proportionality for the similar triangles. Then use the given distances to estimate the radius of the moon.
**Mixed Review**

**Finding Slope** Find the slope of the line that passes through the given points. (Review 3.6 for 8.4)

53. \(A(-1, 4), B(3, 8)\)  
54. \(P(0, -7), Q(-6, -3)\)  
55. \(J(9, 4), K(2, 5)\)  
56. \(L(-2, -3), M(1, 10)\)  
57. \(S(-4, 5), T(-6, -3)\)  
58. \(Y(-1, 6), Z(5, -5)\)

**Finding Angle Measures** Find the value of \(x\). (Review 4.1 for 8.4)

59.  
60.  
61.  

**Solving Proportions** Solve the proportion. (Review 8.1)

62. \(\frac{x}{9} = \frac{6}{27}\)  
63. \(\frac{4}{y} = \frac{2}{19}\)  
64. \(\frac{5}{24} = \frac{25}{z}\)  
65. \(\frac{4}{13} = \frac{b}{8}\)  
66. \(\frac{11}{x + 2} = \frac{9}{x}\)  
67. \(\frac{3x + 7}{5} = \frac{4x}{6}\)

**Quiz 1**

Solve the proportions. (Lesson 8.1)

1. \(\frac{p}{15} = \frac{2}{3}\)  
2. \(\frac{5}{7} = \frac{20}{d}\)  
3. \(\frac{4}{2x - 6} = \frac{16}{x}\)

**Self-Test for Lessons 8.1–8.3**

Find the geometric mean of the two numbers. (Lesson 8.2)

4. 7 and 63  
5. 5 and 11  
6. 10 and 7

In Exercises 7 and 8, the two polygons are similar. Find the value of \(x\). Then find the scale factor and the ratio of the perimeters. (Lesson 8.3)

7. \(x + 1\)  
8. \(x\)

**Comparing Photo Sizes** Use the following information. (Lesson 8.3)

You are ordering your school pictures. You decide to order one \(8 \times 10\) (8 inches by 10 inches), two \(5 \times 7\)’s (5 inches by 7 inches), and 24 wallets \(\left(2\frac{1}{4}\right)\) inches by \(3\frac{1}{4}\) inches.

9. Are any of these sizes similar to each other?

10. Suppose you want the wallet photos to be similar to the \(8 \times 10\) photo. If the wallet photo were \(2\frac{1}{2}\) inches wide, how tall would it be?
Similar Triangles

**GOAL 1** Identifying Similar Triangles

In this lesson, you will continue the study of similar polygons by looking at properties of similar triangles. The activity that follows Example 1 allows you to explore one of these properties.

**EXAMPLE 1** Writing Proportionality Statements

In the diagram, \( \triangle BTW \sim \triangle ETC \).

a. Write the statement of proportionality.

b. Find \( m\angle TEC \).

c. Find \( ET \) and \( BE \).

**SOLUTION**

a. \( \frac{ET}{BT} = \frac{TC}{TW} = \frac{CE}{WB} \)

b. \( \angle B \equiv \angle TEC \), so \( m\angle TEC = 79^\circ \).

c. \( \frac{CE}{WB} = \frac{ET}{BT} \) \hspace{1cm} \text{Write proportion.}

\[
\frac{3}{12} = \frac{ET}{20} \hspace{1cm} \text{Substitute.}
\]

\[
\frac{3(20)}{12} = ET \hspace{1cm} \text{Multiply each side by 20.}
\]

\[
5 = ET \hspace{1cm} \text{Simplify.}
\]

Because \( BE = BT - ET \), \( BE = 20 - 5 = 15 \).

\( \text{So, } ET \text{ is 5 units and } BE \text{ is 15 units.} \)

**ACTIVITY** Developing Concepts

**Investigating Similar Triangles**

Use a protractor and a ruler to draw two noncongruent triangles so that each triangle has a 40° angle and a 60° angle. Check your drawing by measuring the third angle of each triangle—it should be 80°. Why? Measure the lengths of the sides of the triangles and compute the ratios of the lengths of corresponding sides. Are the triangles similar?
**Example 2** Proving that Two Triangles are Similar

Color variations in the tourmaline crystal shown lie along the sides of isosceles triangles. In the triangles each vertex angle measures 52°. Explain why the triangles are similar.

**Solution**

Because the triangles are isosceles, you can determine that each base angle is 64°. Using the AA Similarity Postulate, you can conclude that the triangles are similar.

**Example 3** Why a Line Has Only One Slope

Use properties of similar triangles to explain why any two points on a line can be used to calculate the slope. Find the slope of the line using both pairs of points shown.

**Solution**

By the AA Similarity Postulate $\triangle BEC \sim \triangle AFD$, so the ratios of corresponding sides are the same. In particular, $\frac{CE}{DF} = \frac{BE}{AF}$.

By a property of proportions, $\frac{CE}{BE} = \frac{DF}{AF}$.

The slope of a line is the ratio of the change in $y$ to the corresponding change in $x$. The ratios $\frac{CE}{BE}$ and $\frac{DF}{AF}$ represent the slopes of $BC$ and $AD$, respectively.

Because the two slopes are equal, any two points on a line can be used to calculate its slope. You can verify this with specific values from the diagram.

\[
\text{slope of } BC = \frac{3 - 0}{4 - 2} = \frac{3}{2} \\
\text{slope of } AD = \frac{6 - (-3)}{6 - 0} = \frac{9}{6} = \frac{3}{2}
\]
**Example 4** Using Similar Triangles

**Aerial Photography** Low-level aerial photos can be taken using a remote-controlled camera suspended from a blimp. You want to take an aerial photo that covers a ground distance \( g \) of 50 meters. Use the proportion \( \frac{f}{h} = \frac{n}{g} \) to estimate the altitude \( h \) that the blimp should fly at to take the photo. In the proportion, use \( f = 8 \text{ cm} \) and \( n = 3 \text{ cm} \). These two variables are determined by the type of camera used.

**Solution**

\[
\frac{f}{h} = \frac{n}{g} \\
\frac{8}{h} = \frac{3}{50} \\
3h = 400 \\
h = 133
\]

The blimp should fly at an altitude of about 133 meters to take a photo that covers a ground distance of 50 meters.

---

In Lesson 8.3, you learned that the perimeters of similar polygons are in the same ratio as the lengths of the corresponding sides. This concept can be generalized as follows. If two polygons are similar, then the ratio of any two corresponding lengths (such as altitudes, medians, angle bisector segments, and diagonals) is equal to the scale factor of the similar polygons.

**Example 5** Using Scale Factors

Find the length of the altitude \( QS \).

**Solution**

Find the scale factor of \( \triangle NQP \) to \( \triangle TQR \).

\[
\frac{NP}{TR} = \frac{12 + 12}{8 + 8} = \frac{24}{16} = \frac{3}{2}
\]

Now, because the ratio of the lengths of the altitudes is equal to the scale factor, you can write the following equation.

\[
\frac{QM}{QS} = \frac{3}{2}
\]

Substitute 6 for \( QM \) and solve for \( QS \) to show that \( QS = 4 \).
**GuIdeD PracciCe**

**Vocabulary Check ✓**

1. If \(\triangle ABC \sim \triangle XYZ\), \(AB = 6\), and \(XY = 4\), what is the **scale factor** of the triangles?

**Concept Check ✓**

2. The points \(A(2, 3), B(-1, 6), C(4, 1),\) and \(D(0, 5)\) lie on a line. Which two points could be used to calculate the slope of the line? Explain.

3. Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent?

**Skill Check ✓**

Determine whether \(\triangle CDE \sim \triangle FGH\).

4. 

5. 

In the diagram shown \(\triangle JKL \sim \triangle MNP\).

6. Find \(m\angle J, m\angle N,\) and \(m\angle P\).

7. Find \(MP\) and \(PN\).

8. Given that \(\angle CAB \equiv \angle CBD\), how do you know that \(\triangle ABC \sim \triangle BDC\)? Explain your answer.

**PraCtiCe anD ApPliCatiOns**

**USIng SiMiLarity StaTeMeNts** The triangles shown are similar. List all the pairs of congruent angles and write the statement of proportionality.

9. 

10. 

11. 

**LOGiCAL REAsOniNG** Use the diagram to complete the following.

12. \(\triangle PQR \sim \triangle \) __

13. \(\frac{PQ}{?} = \frac{QR}{?} = \frac{RP}{?}\)

14. \(\frac{20}{?} = \frac{?}{12}\)

15. \(\frac{?}{20} = \frac{18}{?}\)

16. \(y = \) __

17. \(x = \) __
**DETERMINING SIMILARITY** Determine whether the triangles can be proved similar. If they are similar, write a similarity statement. If they are not similar, explain why.

18. 19. 20.

21. 22. 23.


**USING ALGEBRA** Using the labeled points, find the slope of the line. To verify your answer, choose another pair of points and find the slope using the new points. Compare the results.

27. 28.

29. **USING ALGEBRA** Find coordinates for point $E$ so that $\triangle OBC \sim \triangle ODE$.

29. $O(0, 0), B(0, 3), C(6, 0), D(0, 5)$

30. $O(0, 0), B(0, 4), C(3, 0), D(0, 7)$

31. $O(0, 0), B(0, 1), C(5, 0), D(0, 6)$

32. $O(0, 0), B(0, 8), C(4, 0), D(0, 9)$
**USING ALGEBRA** You are given that \(ABCD\) is a trapezoid, \(AB = 8\), \(AE = 6\), \(EC = 15\), and \(DE = 10\).

33. \(\triangle ABE \sim \triangle ?\)  
34. \(\frac{AB}{?} = \frac{AE}{?} = \frac{BE}{?}\)  
35. \(\frac{6}{?} = \frac{8}{?}\)  
36. \(\frac{15}{?} = \frac{10}{?}\)  
37. \(x = ?\)  
38. \(y = ?\)

**SIMILAR TRIANGLES** The triangles are similar. Find the value of the variable.

39.  
![Diagram](image1)

40.  
![Diagram](image2)

41. \(y - 3\)  
42. \(z\)

43. \(45^{\circ}\)  
44. \(35^{\circ}\)

**SIMILAR TRIANGLES** The segments in blue are special segments in the similar triangles. Find the value of the variable.

45.  
![Diagram](image3)

46.  
![Diagram](image4)

47.  
![Diagram](image5)

48. **PROOF** Write a paragraph or two-column proof.
   **GIVEN** ▶ \(KM \perp JL\), \(JK \perp KL\)
   **PROVE** ▶ \(\triangle JKL \sim \triangle JMK\)
49. **PROOF** Write a paragraph proof or a two-column proof. The National Humanities Center is located in Research Triangle Park in North Carolina. Some of its windows consist of nested right triangles, as shown in the diagram. Prove that \( \triangle ABE \sim \triangle CDE \).

**GIVEN**
- \( \angle ECD \) is a right angle,
- \( \angle EAB \) is a right angle.

**PROVE**
- \( \triangle ABE \sim \triangle CDE \)

**LOGICAL REASONING** In Exercises 50–52, decide whether the statement is **true** or **false**. Explain your reasoning.

50. If an acute angle of a right triangle is congruent to an acute angle of another right triangle, then the triangles are similar.

51. Some equilateral triangles are not similar.

52. All isosceles triangles with a 40° vertex angle are similar.

53. **ICE HOCKEY** A hockey player passes the puck to a teammate by bouncing the puck off the wall of the rink as shown. From physics, the angles that the path of the puck makes with the wall are congruent. How far from the wall will the pass be picked up by his teammate?

54. **TECHNOLOGY** Use geometry software to verify that any two points on a line can be used to calculate the slope of the line. Draw a line \( k \) with a negative slope in a coordinate plane. Draw two right triangles of different size whose hypotenuses lie along line \( k \) and whose other sides are parallel to the \( x \)- and \( y \)-axes. Calculate the slope of each triangle by finding the ratio of the vertical side length to the horizontal side length. Are the slopes equal?

55. **THE GREAT PYRAMID** The Greek mathematician Thales (640–546 B.C.) calculated the height of the Great Pyramid in Egypt by placing a rod at the tip of the pyramid’s shadow and using similar triangles.

In the figure, \( PQ \perp QT, SR \perp QT, \) and \( PR \parallel ST \). Write a paragraph proof to show that the height of the pyramid is 480 feet.

56. **ESTIMATING HEIGHT** On a sunny day, use a rod or pole to estimate the height of your school building. Use the method that Thales used to estimate the height of the Great Pyramid in Exercise 55.
57. **MULTI-STEP PROBLEM** Use the following information.

Going from his own house to Raul’s house, Mark drives due south one mile, due east three miles, and due south again three miles. What is the distance between the two houses as the crow flies?

a. Explain how to prove that \( \triangle ABX \sim \triangle DCX \).

b. Use corresponding side lengths of the triangles to calculate \( BX \).

c. Use the Pythagorean Theorem to calculate \( AX \), and then \( DX \). Then find \( AD \).

d. **Writing** Using the properties of rectangles, explain a way that a point \( E \) could be added to the diagram so that \( AD \) would be the hypotenuse of \( \triangle AED \), and \( AE \) and \( ED \) would be its legs of known length.

**HUMAN VISION** In Exercises 58–60, use the following information.

The diagram shows how similar triangles relate to human vision. An image similar to a viewed object appears on the retina. The actual height of the object is proportional to the size of the image as it appears on the retina \( r \). In the same manner, the distances from the object to the lens of the eye \( d \) and from the lens to the retina, 25 mm in the diagram, are also proportional.

58. Write a proportion that relates \( r \), \( d \), \( h \), and 25 mm.

59. An object that is 10 meters away appears on the retina as 1 mm tall. Find the height of the object.

60. An object that is 1 meter tall appears on the retina as 1 mm tall. How far away is the object?

**MIXED REVIEW**

61. **USING THE DISTANCE FORMULA** Find the distance between the points \( A(-17, 12) \) and \( B(14, -21) \). (Review 1.3)

**TRIANGLE MIDSEREGMENTS** \( M, N, \) and \( P \) are the midpoints of the sides of \( \triangle JKL \). Complete the statement. (Review 5.4 for 8.5)

62. \( \overline{NP} \parallel \) ____

63. If \( NP = 23 \), then \( KJ = \) ___.

64. If \( KN = 16 \), then \( MP = \) ___.

65. If \( JL = 24 \), then \( MN = \) ___.

**PROPORTIONS** Solve the proportion. (Review 8.1)

66. \( \frac{x}{12} = \frac{3}{8} \)

67. \( \frac{3}{y} = \frac{12}{32} \)

68. \( \frac{17}{x} = \frac{11}{33} \)

69. \( \frac{34}{11} = \frac{x + 6}{3} \)

70. \( \frac{23}{24} = \frac{x}{72} \)

71. \( \frac{8}{x} = \frac{x}{32} \)
In this lesson, you will study two additional ways to prove that two triangles are similar: the Side-Side-Side (SSS) Similarity Theorem and the Side-Angle-Side (SAS) Similarity Theorem. The first theorem is proved in Example 1 and you are asked to prove the second theorem in Exercise 31.

**Theorem 8.2  Side-Side-Side (SSS) Similarity Theorem**

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

If \( \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \),

then \( \triangle ABC \sim \triangle PQR \).

**Theorem 8.3  Side-Angle-Side (SAS) Similarity Theorem**

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

If \( \angle X \cong \angle M \) and \( \frac{ZX}{PM} = \frac{XY}{MN} \),

then \( \triangle XYZ \sim \triangle MNP \).

**Example 1  Proof of Theorem 8.2**

**Goal** 1  Using Similarity Theorems

**Goal** 2  Use similarity theorems to prove that two triangles are similar.

**Goal** 3  Use similar triangles to solve real-life problems, such as finding the height of a climbing wall in Example 5.

**Why you should learn it**

To solve real-life problems, such as estimating the height of the Unisphere in Ex. 29.

**Theorems**

**Theorem 8.2**  Side-Side-Side (SSS) Similarity Theorem

If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.

If \( \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \),

then \( \triangle ABC \sim \triangle PQR \).

**Theorem 8.3**  Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

If \( \angle X \cong \angle M \) and \( \frac{ZX}{PM} = \frac{XY}{MN} \),

then \( \triangle XYZ \sim \triangle MNP \).

**Solution**

Paragraph Proof  Locate \( P \) on \( RS \) so that \( PS = LM \). Draw \( PQ \) so that \( PQ \parallel RT \).

Then \( \triangle RST \sim \triangle PSQ \), by the AA Similarity Postulate, and \( \frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP} \).

Because \( PS = LM \), you can substitute in the given proportion and find that \( SQ = MN \) and \( QP = NL \). By the SSS Congruence Theorem, it follows that \( \triangle PSQ \equiv \triangle LMN \). Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that \( \triangle RST \sim \triangle LMN \).
EXAMPLE 2 Using the SSS Similarity Theorem

Which of the following three triangles are similar?

\[ \triangle ABC \quad \triangle DEF \quad \triangle GHJ \]

**SOLUTION**

To decide which, if any, of the triangles are similar, you need to consider the ratios of the lengths of corresponding sides.

**Ratios of Side Lengths of \( \triangle ABC \) and \( \triangle DEF \)**

\[
\frac{AB}{DE} = \frac{6}{4} = \frac{3}{2}, \quad \frac{CA}{FD} = \frac{12}{8} = \frac{3}{2}, \quad \frac{BC}{EF} = \frac{9}{6} = \frac{3}{2}
\]

- **Shortest sides**
- **Longest sides**
- **Remaining sides**

Because all of the ratios are equal, \( \triangle ABC \sim \triangle DEF \).

**Ratios of Side Lengths of \( \triangle ABC \) and \( \triangle GHJ \)**

\[
\frac{AB}{GH} = \frac{6}{6} = 1, \quad \frac{CA}{JG} = \frac{12}{14} = \frac{6}{7}, \quad \frac{BC}{HJ} = \frac{9}{10}
\]

- **Shortest sides**
- **Longest sides**
- **Remaining sides**

Because the ratios are not equal, \( \triangle ABC \) and \( \triangle GHJ \) are not similar.

Since \( \triangle ABC \) is similar to \( \triangle DEF \) and \( \triangle ABC \) is not similar to \( \triangle GHJ \), \( \triangle DEF \) is not similar to \( \triangle GHJ \).

EXAMPLE 3 Using the SAS Similarity Theorem

Use the given lengths to prove that \( \triangle RST \sim \triangle PSQ \).

**SOLUTION**

**GIVEN** \( SP = 4, PR = 12, SQ = 5, QT = 15 \)

**PROVE** \( \triangle RST \sim \triangle PSQ \)

**Paragraph Proof** Use the SAS Similarity Theorem. Begin by finding the ratios of the lengths of the corresponding sides.

\[
\frac{SR}{SP} = \frac{SP + PR}{SP} = \frac{4 + 12}{4} = \frac{16}{4} = 4 \quad \text{and} \quad \frac{ST}{SQ} = \frac{SQ + QT}{SQ} = \frac{5 + 15}{5} = \frac{20}{5} = 4
\]

So, the lengths of sides \( SR \) and \( ST \) are proportional to the lengths of the corresponding sides of \( \triangle PSQ \). Because \( \angle S \) is the included angle in both triangles, use the SAS Similarity Theorem to conclude that \( \triangle RST \sim \triangle PSQ \).
**Example 4 Using a Pantograph**

**SCALE DRAWING** As you move the tracing pin of a pantograph along a figure, the pencil attached to the far end draws an enlargement. As the pantograph expands and contracts, the three brads and the tracing pin always form the vertices of a parallelogram. The ratio of $PR$ to $PT$ is always equal to the ratio of $PQ$ to $PS$. Also, the suction cup, the tracing pin, and the pencil remain collinear.

a. How can you show that $\triangle PRQ \sim \triangle PTS$?

b. In the diagram, $PR$ is 10 inches and $RT$ is 10 inches. The length of the cat, $RQ$, in the original print is 2.4 inches. Find the length $TS$ in the enlargement.

**Solution**

a. You know that $\frac{PR}{PT} = \frac{PQ}{PS}$. Because $\angle P \cong \angle P$, you can apply the SAS Similarity Theorem to conclude that $\triangle PRQ \sim \triangle PTS$.

b. Because the triangles are similar, you can set up a proportion to find the length of the cat in the enlarged drawing.

$$\frac{PR}{PT} = \frac{RQ}{TS}$$  \hspace{1cm} \text{Write proportion.}

$$\frac{10}{20} = \frac{2.4}{TS}$$  \hspace{1cm} \text{Substitute.}

$$TS = 4.8$$  \hspace{1cm} \text{Solve for } TS.

So, the length of the cat in the enlarged drawing is 4.8 inches.

---

Similar triangles can be used to find distances that are difficult to measure directly. One technique is called Thales’ shadow method (page 486), named after the Greek geometer Thales who used it to calculate the height of the Great Pyramid.
EXAMPLE 5  Finding Distance Indirectly

ROCK CLIMBING  You are at an indoor climbing wall. To estimate the height of the wall, you place a mirror on the floor 85 feet from the base of the wall. Then you walk backward until you can see the top of the wall centered in the mirror. You are 6.5 feet from the mirror and your eyes are 5 feet above the ground. Use similar triangles to estimate the height of the wall.

SOLUTION

Due to the reflective property of mirrors, you can reason that $\triangle ACB \cong \triangle ECD$.

Using the fact that $\triangle ABC$ and $\triangle EDC$ are right triangles, you can apply the AA Similarity Postulate to conclude that these two triangles are similar.

$$\frac{DE}{BA} = \frac{EC}{AC}$$  
Ratios of lengths of corresponding sides are equal.

$$\frac{DE}{5} = \frac{85}{6.5}$$  
Substitute.

$$65.38 \approx DE$$  
Multiply each side by 5 and simplify.

So, the height of the wall is about 65 feet.

EXAMPLE 6  Finding Distance Indirectly

INDIRECT MEASUREMENT  To measure the width of a river, you use a surveying technique, as shown in the diagram. Use the given lengths (measured in feet) to find $RQ$.

SOLUTION

By the AA Similarity Postulate, $\triangle PQR \sim \triangle STR$.

$$\frac{RQ}{RT} = \frac{PQ}{ST}$$  
Write proportion.

$$\frac{RQ}{12} = \frac{63}{9}$$  
Substitute.

$$RQ = 12 \cdot 7$$  
Multiply each side by 12.

$$RQ = 84$$  
Simplify.

So, the river is 84 feet wide.
1. You want to prove that \( \triangle FHG \) is similar to \( \triangle RXS \) by the SSS Similarity Theorem. Complete the proportion that is needed to use this theorem.

\[
\frac{FH}{?} = \frac{?}{XS} = \frac{FG}{?}
\]

2. Name a postulate or theorem that can be used to prove that the two triangles are similar. Then, write a similarity statement.

3. Which triangles are similar to \( \triangle ABC \)? Explain.

4. The side lengths of \( \triangle ABC \) are 2, 5, and 6, and \( \triangle DEF \) has side lengths of 12, 30, and 36. Find the ratios of the lengths of the corresponding sides of \( \triangle ABC \) to \( \triangle DEF \). Are the two triangles similar? Explain.

**Determing Similarity** In Exercises 6–8, determine which two of the three given triangles are similar. Find the scale factor for the pair.

6. 

7. 

8. 

**Extra Practice** to help you master skills is on p. 818.
**DETERMINING SIMILARITY** Are the triangles similar? If so, state the similarity and the postulate or theorem that justifies your answer.

9. \[ \triangle JKL \]
   \[ \triangle ABC \]
   - \[ \triangle JKL \] and \[ \triangle ABC \] are similar by the **SAS** postulate.

10. \[ \triangle XYZ \]
    \[ \triangle PQR \]
    - \[ \triangle XYZ \] and \[ \triangle PQR \] are similar by the **AA** criterion.

11. \[ \triangle ABD \]
    \[ \triangle CDE \]
    - \[ \triangle ABD \] and \[ \triangle CDE \] are similar by the **SAS** postulate.

12. \[ \triangle DEF \]
    \[ \triangle GHI \]
    - \[ \triangle DEF \] and \[ \triangle GHI \] are similar by the **SAS** postulate.

13. \[ \triangle RQS \]
    \[ \triangle EFD \]
    - \[ \triangle RQS \] and \[ \triangle EFD \] are similar by the **SAS** postulate.

14. \[ \triangle WXY \]
    \[ \triangle ZYX \]
    - \[ \triangle WXY \] and \[ \triangle ZYX \] are similar by the **SAS** postulate.

**LOGICAL REASONING** Draw the given triangles roughly to scale. Then, name a postulate or theorem that can be used to prove that the triangles are similar.

15. The side lengths of \( \triangle PQR \) are 16, 8, and 18, and the side lengths of \( \triangle XYZ \) are 9, 8, and 4.

16. In \( \triangle ABC \), \( \angle A = 28^\circ \) and \( \angle B = 62^\circ \). In \( \triangle DEF \), \( \angle D = 28^\circ \) and \( \angle F = 90^\circ \).

17. In \( \triangle STU \), the length of \( SU \) is 24, and the length of \( SU \) is 18, and \( \angle S = 65^\circ \). The length of \( JK \) is 6, \( \angle J = 65^\circ \), and the length of \( JL \) is 8 in \( \triangle JKL \).

18. The ratio of \( VW \) to \( MN \) is 6 to 1. In \( \triangle VWX \), \( \angle W = 30^\circ \), and in \( \triangle MNP \), \( \angle N = 30^\circ \). The ratio of \( WX \) to \( NP \) is 6 to 1.

**FINDING MEASURES AND LENGTHS** Use the diagram shown to complete the statements.

19. \( \angle CED = \) ?
20. \( \angle EDC = \) ?
21. \( \angle DCE = \) ?
22. \( FC = \) ?
23. \( EC = \) ?
24. \( DE = \) ?
25. \( CB = \) ?

26. Name the three pairs of triangles that are similar in the figure.
**DETERMINING SIMILARITY** Determine whether the triangles are similar. If they are, write a similarity statement and solve for the variable.

27. 

28. 

29. **UNISPHERE** You are visiting the Unisphere at Flushing Meadow Park in New York. To estimate the height of the stainless steel model of Earth, you place a mirror on the ground and stand where you can see the top of the model in the mirror. Use the diagram shown to estimate the height of the model.

30. **PARAGRAPH PROOF** Two isosceles triangles are similar if the vertex angle of one triangle is congruent to the vertex angle of the other triangle. Write a paragraph proof of this statement and include a labeled figure.

31. **PARAGRAPH PROOF** Write a paragraph proof of Theorem 8.3.

**FINDING DISTANCES INDIRECTLY** Find the distance labeled x.

32. 

33. 

**FLAGPOLE HEIGHT** In Exercises 34 and 35, use the following information.

Julia uses the shadow of the flagpole to estimate its height. She stands so that the tip of her shadow coincides with the tip of the flagpole’s shadow as shown. Julia is 5 feet tall. The distance from the flagpole to Julia is 28 feet and the distance between the tip of the shadows and Julia is 7 feet.

34. Calculate the height of the flagpole.

35. Explain why Julia’s shadow method works.
QUANTITATIVE COMPARISON  In Exercises 36 and 37, use the diagram, in which \( \triangle ABC \sim \triangle XYZ \), and the ratio \( AB:XY \) is 2:5. Choose the statement that is true about the given quantities.

- A The quantity in column A is greater.
- B The quantity in column B is greater.
- C The two quantities are equal.
- D The relationship cannot be determined from the given information.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>36. The perimeter of ( \triangle ABC )</td>
<td>The length ( XY )</td>
</tr>
<tr>
<td>37. The distance ( XY + BC )</td>
<td>The distance ( XZ + YZ )</td>
</tr>
</tbody>
</table>

38. **DESIGNING THE LOOP**
A portion of an amusement park ride called the Loop is shown. Find the length of \( EF \). (Hint: Use similar triangles.)

MIXED REVIEW

**ANALYZING ANGLE BISECTORS** \( \overline{BD} \) is the angle bisector of \( \angle ABC \). Find any angle measures not given in the diagram. (Review 1.5 for 8.6)

39. 40. 41.

**RECOGNIZING ANGLES** Use the diagram shown to complete the statement. (Review 3.1 for 8.6)

42. \( \angle 5 \) and \( \angle ? \) are alternate exterior angles.
43. \( \angle 8 \) and \( \angle ? \) are consecutive interior angles.
44. \( \angle 10 \) and \( \angle ? \) are alternate interior angles.
45. \( \angle 9 \) and \( \angle ? \) are corresponding angles.

**FINDING COORDINATES** Find the coordinates of the image after the reflection without using a coordinate plane. (Review 7.2)

46. \( T(0, 5) \) reflected in the x-axis
47. \( P(-2, 7) \) reflected in the y-axis
48. \( B(-3, -10) \) reflected in the y-axis
49. \( C(-5, -1) \) reflected in the x-axis
Quiz 2

Determine whether you can show that the triangles are similar. State any angle measures that are not given. (Lesson 8.4)

1.

2.

3.

In Exercises 4–6, you are given the ratios of the lengths of the sides of \( \triangle DEF \). If \( \triangle ABC \) has sides of lengths 3, 6, and 7 units, are the triangles similar? (Lesson 8.5)

4. 4:7:8

5. 6:12:14

6. 1:2:7/3

7. **Distance Across Water**

Use the known distances in the diagram to find the distance across the lake from \( A \) to \( B \). (Lesson 8.5)

---

**Math & History**

**The Golden Rectangle**

**Then**

Thousands of years ago, Greek mathematicians became interested in the golden ratio, a ratio of about 1:1.618. A rectangle whose side lengths are in the golden ratio is called a golden rectangle. Such rectangles are believed to be especially pleasing to look at.

**Now**

The golden ratio has been found in the proportions of many works of art and architecture, including the works shown in the timeline below.

1. Follow the steps below to construct a golden rectangle. When you are done, check to see whether the ratio of the width to the length is 1:1.618.
   - Construct a square. Mark the midpoint \( M \) of the bottom side.
   - Place the compass point at \( M \) and draw an arc through the upper right corner of the square.
   - Extend the bottom side of the square to intersect with the arc. The intersection point is the corner of a golden rectangle. Complete the rectangle.

---

**APPLICATION LINK**

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The Osirion (underground Egyptian temple)

The Parthenon, Athens, Greece


Le Corbusier uses golden ratios based on this human figure in his architecture.
Proportions and Similar Triangles

**GOAL 1** **Using Proportionality Theorems**

In this lesson, you will study four proportionality theorems. Similar triangles are used to prove each theorem. You are asked to prove the theorems in Exercises 31–33 and 38.

**Theorems**

**Theorem 8.4** *Triangle Proportionality Theorem*

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

If $TU \parallel QS$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

**Theorem 8.5** *Converse of the Triangle Proportionality Theorem*

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

If $\frac{RT}{TQ} = \frac{RU}{US}$, then $TU \parallel QS$.

**Example 1** *Finding the Length of a Segment*

In the diagram $AB \parallel ED$,

$BD = 8$, $DC = 4$, and $AE = 12$.

What is the length of $EC$?

**Solution**

$\frac{DC}{BD} = \frac{EC}{AE}$  
$\frac{4}{8} = \frac{EC}{12}$  
$\frac{4(12)}{8} = EC$  
$6 = EC$  

So, the length of $EC$ is 6.
**EXAMPLE 2  Determining Parallels**

Given the diagram, determine whether $MN \parallel GH$.

**SOLUTION**

Begin by finding and simplifying the ratios of the two sides divided by $MN$.

\[
\frac{LM}{MG} = \frac{56}{21} = \frac{8}{3} \quad \text{and} \quad \frac{LN}{NH} = \frac{48}{16} = \frac{3}{1}.
\]

Because $\frac{8}{3} \neq \frac{3}{1}$, $MN$ is not parallel to $GH$.

**THEOREMS**

**THEOREM 8.6**

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

If $r \parallel s$ and $s \parallel t$, and $\ell$ and $m$ intersect $r$, $s$, and $t$, then $\frac{UW}{WY} = \frac{VX}{XZ}$.

**THEOREM 8.7**

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

If $CD \parallel ACB$, then $\frac{AD}{DB} = \frac{CA}{CB}$.

**EXAMPLE 3  Using Proportionality Theorems**

In the diagram, $\angle 1 \equiv \angle 2 \equiv \angle 3$, and $PQ = 9$, $QR = 15$, and $ST = 11$. What is the length of $TU$?

**SOLUTION**

Because corresponding angles are congruent the lines are parallel and you can use Theorem 8.6.

\[
\frac{PQ}{QR} = \frac{ST}{TU} \quad \text{Parallel lines divide transversals proportionally.}
\]

\[
\frac{9}{15} = \frac{11}{TU} \quad \text{Substitute.}
\]

\[
9 \cdot TU = 15 \cdot 11 \quad \text{Cross product property}
\]

\[
TU = \frac{15(11)}{9} = \frac{55}{3} \quad \text{Divide each side by 9 and simplify.}
\]

So, the length of $TU$ is $\frac{55}{3}$, or $18\frac{1}{3}$.
EXAMPLE 4  Using Proportionality Theorems

In the diagram, \( \angle CAD \equiv \angle DAB \). Use the given side lengths to find the length of \( DC \).

**SOLUTION**

Since \( AD \) is an angle bisector of \( \angle CAB \), you can apply Theorem 8.7.

Let \( x = DC \). Then, \( BD = 14 - x \).

\[
\frac{AB}{AC} = \frac{BD}{DC}
\]

Apply Theorem 8.7.

\[
\frac{9}{15} = \frac{14 - x}{x}
\]

Substitute.

\[9 \cdot x = 15(14 - x)\]

Cross product property

\[9x = 210 - 15x\]

Distributive property

\[24x = 210\]

Add \( 15x \) to each side.

\[x = 8.75\]

Divide each side by 24.

So, the length of \( DC \) is 8.75 units.

**EXAMPLE 4** Using Proportionality Theorems

**SOLUTION**

Since \( AD \) is an angle bisector of \( \angle CAB \), you can apply Theorem 8.7.

Let \( x = DC \). Then, \( BD = 14 - x \).

\[
\frac{AB}{AC} = \frac{BD}{DC}
\]

Apply Theorem 8.7.

\[
\frac{9}{15} = \frac{14 - x}{x}
\]

Substitute.

\[9 \cdot x = 15(14 - x)\]

Cross product property

\[9x = 210 - 15x\]

Distributive property

\[24x = 210\]

Add \( 15x \) to each side.

\[x = 8.75\]

Divide each side by 24.

So, the length of \( DC \) is 8.75 units.

**ACTIVITY**

**Construction**

Dividing a Segment into Equal Parts (4 shown)

1. Draw a line segment that is about 3 inches long. Label the endpoints \( A \) and \( B \). Choose any point \( C \) not on \( AB \). Draw \( AC \).

2. Using any length, place the compass point at \( A \) and make an arc intersecting \( AC \) at \( D \).

3. Using the same compass setting, make additional arcs on \( AC \). Label the points \( E, F, \) and \( G \) so that \( AD = DE = EF = FG \).

4. Draw \( GB \). Construct a line parallel to \( GB \) through \( D \). Continue constructing parallel lines and label the points as shown. Explain why \( AJ = JK = KL = LB \).
GOAL 2  USING PROPORTIONALITY THEOREMS IN REAL LIFE

EXAMPLE 5  Finding the Length of a Segment

BUILDING CONSTRUCTION  You are insulating your attic, as shown. The vertical 2 × 4 studs are evenly spaced. Explain why the diagonal cuts at the tops of the strips of insulation should have the same lengths.

SOLUTION
Because the studs $AD, BE,$ and $CF$ are each vertical, you know that they are parallel to each other. Using Theorem 8.6, you can conclude that $\frac{DE}{EF} = \frac{AB}{BC}$.
Because the studs are evenly spaced, you know that $DE = EF$. So, you can conclude that $AB = BC$, which means that the diagonal cuts at the tops of the strips have the same lengths.

EXAMPLE 6  Finding Segment Lengths

In the diagram $KL \parallel MN$. Find the values of the variables.

SOLUTION
To find the value of $x$, you can set up a proportion.

$\frac{9}{13.5} = \frac{37.5 - x}{x}$  \hspace{1cm} Write proportion.

$13.5(37.5 - x) = 9x$  \hspace{1cm} Cross product property

$506.25 - 13.5x = 9x$  \hspace{1cm} Distributive property

$506.25 = 22.5x$  \hspace{1cm} Add 13.5x to each side.

$22.5 = x$  \hspace{1cm} Divide each side by 22.5.

Since $KL \parallel MN$, $\triangle JKL \sim \triangle JMN$ and $\frac{JK}{JM} = \frac{KL}{MN}$.

$\frac{9}{13.5 + 9} = \frac{7.5}{y}$  \hspace{1cm} Write proportion.

$9y = 7.5(22.5)$  \hspace{1cm} Cross product property

$y = 18.75$  \hspace{1cm} Divide each side by 9.
1. Complete the following: If a line divides two sides of a triangle proportionally, then it is __ to the third side. This theorem is known as the __.

2. In \( \triangle ABC \), \( \overline{AR} \) bisects \( \angle CAB \). Write the proportionality statement for the triangle that is based on Theorem 8.7.

Determine whether the statement is true or false. Explain your reasoning.

3. \( \frac{FE}{ED} = \frac{FG}{GH} \)

4. \( \frac{FE}{FD} = \frac{FG}{FH} \)

5. \( \frac{EG}{DH} = \frac{EF}{DF} \)

6. \( \frac{ED}{FE} = \frac{EG}{DH} \)

Use the figure to complete the proportion.

7. \( \frac{BD}{BF} = \frac{?}{CG} \)

8. \( \frac{AE}{CE} = \frac{?}{BD} \)

9. \( \frac{?}{GA} = \frac{FD}{FA} \)

10. \( \frac{GA}{?} = \frac{FA}{DA} \)

LOGICAL REASONING

Determine whether the given information implies that \( QS \parallel PT \). Explain.

11.

12.

LOGICAL REASONING

Use the diagram shown to decide if you are given enough information to conclude that \( LP \parallel MQ \). If so, state the reason.

13.

14.

15. \( \frac{NM}{ML} = \frac{NQ}{QP} \)

16. \( \angle MNQ \equiv \angle LNP \)

17. \( \triangle NLP \equiv \triangle NMQ \)

18. \( \angle MQN \equiv \angle LPN \)

19. \( \frac{LM}{MN} = \frac{LP}{MQ} \)

20. \( \triangle LPN \sim \triangle MQN \)
8.6 Proportions and Similar Triangles

USING PROPORTIONALITY THEOREMS

Find the value of the variable.

21. \[ \frac{9}{a} = \frac{15}{5} \]

22. \[ \frac{20}{24} = \frac{12}{c} \]

23. \[ \frac{8}{x} = \frac{20}{15} \]

24. \[ \frac{25}{z} = \frac{8}{12} \]

USING ALGEBRA

Find the value of the variable.

25. \[ \frac{7}{p} = \frac{12}{24} \]

26. \[ \frac{17.5}{q} = \frac{21}{33} \]

27. \[ \frac{6}{f} = \frac{21}{15} \]

28. \[ \frac{14}{1.25g} = \frac{17.5}{7.5} \]

LOT PRICES

The real estate term for the distance along the edge of a piece of property that touches the ocean is “ocean frontage.”

29. Find the ocean frontage (to the nearest tenth of a meter) for each lot shown.

30. CRITICAL THINKING

In general, the more ocean frontage a lot has, the higher its selling price. Which of the lots should be listed for the highest price?
31. **TWO-COLUMN PROOF** Use the diagram shown to write a two-column proof of Theorem 8.4.

**GIVEN** \( DE \parallel AC \)

**PROVE** \( \frac{DA}{BD} = \frac{EC}{BE} \)

32. **PARAGRAPH PROOF** Use the diagram with the auxiliary line drawn to write a paragraph proof of Theorem 8.6.

**GIVEN** \( k_1 \parallel k_2, k_2 \parallel k_3 \)

**PROVE** \( \frac{CB}{BA} = \frac{DE}{EF} \)

33. **PARAGRAPH PROOF** Use the diagram with the auxiliary lines drawn to write a paragraph proof of Theorem 8.7.

**GIVEN** \( \angle YXW \equiv \angle WXZ \)

**PROVE** \( \frac{YW}{WZ} = \frac{XY}{XZ} \)

**FINDING SEGMENT LENGTHS** Use the diagram to determine the lengths of the missing segments.

34. \( A \quad B \quad 11.9 \quad D \quad F \quad 3.5 \quad H \)

35. \( K \quad L \quad 18 \quad N \quad 18 \quad O \quad U \)

**NEW YORK CITY** Use the following information and the map of New York City.

On Fifth Avenue, the distance between E 33rd Street and E 24th Street is about 2600 feet. The distance between those same streets on Broadway is about 2800 feet. All numbered streets are parallel.

36. On Fifth Avenue, the distance between E 24th Street and E 29th Street is about 1300 feet. What is the distance between these two streets on Broadway?

37. On Broadway, the distance between E 33rd Street and E 30th Street is about 1120 feet. What is the distance between these two streets on Fifth Avenue?
38. **Writing** Use the diagram given for the proof of Theorem 8.4 from Exercise 31 to explain how you can prove the Triangle Proportionality Converse, Theorem 8.5.

39. **MULTI-STEP PROBLEM** Use the diagram shown.
   a. If $DB = 6$, $AD = 2$, and $CB = 20$, find $EB$.
   b. Use the diagram to state three correct proportions.
   c. If $DB = 4$, $AB = 10$, and $CB = 20$, find $CE$.
   d. **Writing** Explain how you know that $\triangle ABC$ is similar to $\triangle DBE$.

40. **CONSTRUCTION** Perform the following construction.

   **GIVEN** Segments with lengths $x$, $y$, and $z$

   **CONSTRUCT** A segment of length $p$, such that $\frac{x}{y} = \frac{z}{p}$.

   *(Hint: This construction is like the construction on page 500.)*

---

**MIXED REVIEW**

**USING THE DISTANCE FORMULA** Find the distance between the two points. *(Review 1.3)*

41. $A(10, 5)$  
   $B(−6, −4)$  
42. $A(7, −3)$  
   $B(−9, 4)$  
43. $A(−1, −9)$  
   $B(6, −2)$  
44. $A(0, 11)$  
   $B(−5, 2)$  
45. $A(0, −10)$  
   $B(4, 7)$  
46. $A(8, −5)$  
   $B(0, 4)$

**USING THE DISTANCE FORMULA** Place the figure in a coordinate plane and find the requested information. *(Review 4.7)*

47. Draw a right triangle with legs of 12 units and 9 units. Find the length of the hypotenuse.

48. Draw a rectangle with length 16 units and width 12 units. Find the length of a diagonal.

49. Draw an isosceles right triangle with legs of 6 units. Find the length of the hypotenuse.

50. Draw an isosceles triangle with base of 16 units and height of 6 units. Find the length of the legs.

**TRANSFORMATIONS** Name the type of transformation. *(Review 7.1–7.3, 7.5 for 8.7)*

51.  
52.  
53.
Chapter 8

8.7

Dilations

**GOAL 1** IDENTIFYING DILATIONS

In Chapter 7, you studied rigid transformations, in which the image and preimage of a figure are congruent. In this lesson, you will study a type of nonrigid transformation called a dilation, in which the image and preimage of a figure are similar.

A dilation with center \( C \) and scale factor \( k \) is a transformation that maps every point \( P \) in the plane to a point \( P' \) so that the following properties are true.

1. If \( P \) is not the center point \( C \), then the image point \( P' \) lies on \( CP \). The scale factor \( k \) is a positive number such that \( k = \frac{PC'}{PC} \), and \( k \neq 1 \).
2. If \( P \) is the center point \( C \), then \( P = P' \).

The dilation is a reduction if \( 0 < k < 1 \) and it is an enlargement if \( k > 1 \).

![Diagrams showing dilations]

Reduction: \( k = \frac{CP'}{CP} = \frac{3}{6} = \frac{1}{2} \)

Enlargement: \( k = \frac{CP'}{CP} = \frac{5}{2} \)

Because \( \triangle PQR \sim \triangle P'R'Q' \), \( \frac{P'Q'}{PQ} \) is equal to the scale factor of the dilation.

**EXAMPLE 1** Identifying Dilations

Identify the dilation and find its scale factor.

**a.**

![Diagram showing dilation]

**b.**

![Diagram showing dilation]

**SOLUTION**

a. Because \( \frac{CP'}{CP} = \frac{2}{3} \), the scale factor is \( k = \frac{2}{3} \). This is a reduction.

b. Because \( \frac{CP'}{CP} = \frac{2}{1} \), the scale factor is \( k = 2 \). This is an enlargement.

**STUDENT HELP**

Look Back

For help with the blue to red color scheme used in transformations, see p. 396.
In a coordinate plane, dilations whose centers are the origin have the property that the image of \( P(x, y) \) is \( P'(kx, ky) \).

**EXAMPLE 2  Dilation in a Coordinate Plane**

Draw a dilation of rectangle \( ABCD \) with \( A(2, 2), B(6, 2), C(6, 4), \) and \( D(2, 4) \). Use the origin as the center and use a scale factor of \( \frac{1}{2} \). How does the perimeter of the preimage compare to the perimeter of the image?

**SOLUTION**

Because the center of the dilation is the origin, you can find the image of each vertex by multiplying its coordinates by the scale factor.

\[
\begin{align*}
A(2, 2) & \rightarrow A'(1, 1) \\
B(6, 2) & \rightarrow B'(3, 1) \\
C(6, 4) & \rightarrow C'(3, 2) \\
D(2, 4) & \rightarrow D'(1, 2)
\end{align*}
\]

From the graph, you can see that the preimage has a perimeter of 12 and the image has a perimeter of 6. A preimage and its image after a dilation are similar figures. Therefore, the ratio of the perimeters of a preimage and its image is equal to the scale factor of the dilation.

**ACTIVITY CONSTRUCTION DRAWING A DILATION**

In the construction above, notice that \( \triangle PQR \sim \triangle P'Q'R' \). You can prove this by using the SAS and SSS Similarity Theorems.


**GOAL 2  USING DILATIONS IN REAL LIFE**

**EXAMPLE 3  Finding the Scale Factor**

**SHADOW PUPPETS** Shadow puppets have been used in many countries for hundreds of years. A flat figure is held between a light and a screen. The audience on the other side of the screen sees the puppet's shadow. The shadow is a dilation, or enlargement, of the shadow puppet. When looking at a cross sectional view, \( \triangle LCP \sim \triangle LSH \).

The shadow puppet shown is 12 inches tall (\( CP \) in the diagram). Find the height of the shadow, \( SH \), for each distance from the screen. In each case, by what percent is the shadow larger than the puppet?

a. \( LC = LP = 59 \text{ in.}; LS = LH = 74 \text{ in.} \)

b. \( LC = LP = 66 \text{ in.}; LS = LH = 74 \text{ in.} \)

**SOLUTION**

a. 

\[
\frac{59}{74} = \frac{12}{SH} \quad \frac{LC}{LS} = \frac{CP}{SH}
\]

\[59(15) = 888 \]

\[SH = 15 \text{ inches} \]

To find the percent of size increase, use the scale factor of the dilation.

\[
\text{scale factor} = \frac{SH}{CP}
\]

\[
\frac{15}{12} = 1.25
\]

So, the shadow is 25% larger than the puppet.

b. 

\[
\frac{66}{74} = \frac{12}{SH} \]

\[66(13.45) = 888 \]

\[SH = 13.45 \text{ inches} \]

Use the scale factor again to find the percent of size increase.

\[
\text{scale factor} = \frac{SH}{CP}
\]

\[
\frac{13.45}{12} = 1.12
\]

So, the shadow is about 12% larger than the puppet.

Notice that as the puppet moves closer to the screen, the shadow height decreases.
8.7 Dilations

**Guided Practice**

**Vocabulary Check ✓**
1. In a **dilation** every image is ? to its preimage.

2. **ERROR ANALYSIS** Katie found the scale factor of the dilation shown to be \( \frac{1}{2} \). What did Katie do wrong?

**Concept Check ✓**
3. Is the dilation shown a reduction or an enlargement? How do you know?

4. \( \triangle PQR \) is mapped onto \( \triangle P'Q'R' \) by a dilation with center \( C \). Complete the statement.

5. If \( \frac{CP'}{CP} = \frac{4}{3} \), then \( \triangle P'Q'R' \) is (larger, smaller) than \( \triangle PQR \), and the dilation is (a reduction, an enlargement).

**Skill Check ✓**

Use the following information to draw a dilation of rectangle \( ABCD \).

6. Draw a dilation of rectangle \( ABCD \) on a coordinate plane, with \( A(3, 1) \), \( B(3, 2.5) \), \( C(5, 2.5) \), and \( D(5, 1) \). Use the origin as the center and use a scale factor of 2.

7. Is \( ABCD \sim A'B'C'D' \)? Explain your answer.

**Practice and Applications**

**Identifying Dilations** Identify the dilation and find its scale factor.

8. \[ \triangle PQR \]

9. \[ \triangle PQR \]

**Finding Scale Factors** Identify the dilation, and find its scale factor. Then, find the values of the variables.

10. \[ \triangle PQR \]

11. \[ \triangle PQR \]
DILATIONS IN A COORDINATE PLANE  Use the origin as the center of the dilution and the given scale factor to find the coordinates of the vertices of the image of the polygon.

12. \( k = \frac{1}{2} \)

13. \( k = 2 \)

14. \( k = \frac{1}{3} \)

15. \( k = 4 \)

16. COMPARING RATIOS  Use the triangle shown. Let \( P \) and \( Q \) be the midpoints of the sides \( EG \) and \( FG \), respectively. Find the scale factor and the center of the dilation that enlarges \( \triangle PQG \) to \( \triangle EFG \). Find the ratio of \( EF \) to \( PQ \). How does this ratio compare to the scale factor?

CONSTRUCTION  Copy \( \triangle DEF \) and points \( G \) and \( H \) as shown. Then, use a straightedge and a compass to construct the dilation.

17. \( k = 3 \); Center: \( G \)

18. \( k = \frac{1}{2} \); Center: \( H \)

19. \( k = 2 \); Center: \( E \)

SIMILAR TRIANGLES  The red triangle is the image of the blue triangle after a dilation. Find the values of the variables. Then find the ratio of their perimeters.

20. 

21. 
**IDENTIFYING DILATIONS** \( \triangle ABC \) is mapped onto \( \triangle A'B'C' \) by a dilation. Use the given information to sketch the dilation, identify it as a reduction or an enlargement, and find the scale factor. Then find the missing lengths.

22. In \( \triangle ABC \), \( AB = 6 \), \( BC = 9 \), and \( AC = 12 \). In \( \triangle A'B'C' \), \( A'B' = 2 \). Find the lengths of \( B'C' \) and \( A'C' \).

23. In \( \triangle ABC \), \( AB = 5 \) and \( BC = 7 \). In \( \triangle A'B'C' \), \( A'B' = 20 \) and \( A'C' = 36 \). Find the lengths of \( AC \) and \( B'C' \).

**FLASHLIGHT IMAGE** In Exercises 24–26, use the following information.

You are projecting images onto a wall with a flashlight. The lamp of the flashlight is 8.3 centimeters away from the wall. The preimage is imprinted onto a clear cap that fits over the end of the flashlight. This cap has a diameter of 3 centimeters. The preimage has a height of 2 centimeters, and the lamp of the flashlight is located 2.7 centimeters from the preimage.

24. Sketch a diagram of the dilation.

25. Find the diameter of the circle of light projected onto the wall from the flashlight.

26. Find the height of the image projected onto the wall.

**ENLARGEMENTS** In Exercises 27 and 28, use the following information.

By adjusting the distance between the negative and the enlarged print in the photographic enlarger shown, you can make prints of different sizes. In the diagram shown, you want the enlarged print to be 7 inches wide (A'B'). The negative is 1 inch wide (AB), and the distance between the light source and the negative is 1.25 inches (CD).

27. What is the scale factor of the enlargement?

28. What is the distance between the light source and the enlarged print?

**DIMENSIONS OF PHOTOS** Use the diagram from Exercise 27 to determine the missing information.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>CD</td>
<td>CD'</td>
<td>AB</td>
<td>A'B'</td>
<td></td>
</tr>
<tr>
<td>29.</td>
<td>1.2 in.</td>
<td>7.2 in.</td>
<td>0.8 in.</td>
<td>?</td>
</tr>
<tr>
<td>30.</td>
<td>?</td>
<td>14 cm</td>
<td>2 cm</td>
<td>12 cm</td>
</tr>
<tr>
<td>31.</td>
<td>2 in.</td>
<td>10 in.</td>
<td>?</td>
<td>8.5 in.</td>
</tr>
</tbody>
</table>
32. **Logical Reasoning** Draw any triangle, and label it \( \triangle PQR \). Using a scale factor of 2, draw the image of \( \triangle PQR \) after a dilation with a center outside the triangle, with a center inside the triangle, and with a center on the triangle. Explain the relationship between the three images created.

33. **Writing** Use the information about shadow puppet theaters from Example 3, page 508. Explain how you could use a shadow puppet theater to help another student understand the terms *image*, *preimage*, *center of dilation*, and *dilation*. Draw a diagram and label the terms on the diagram.

34. **Perspective Drawing** Create a perspective drawing by following the given steps.

1. Draw a horizontal line across the paper, and choose a point on this line to be the center of the dilation, also called the *vanishing point*. Next, draw a polygon.
2. Draw rays from the vanishing point to all vertices of the polygon. Draw a reduction of the polygon by locating image points on the rays.
3. Connect the preimage to the image by darkening the segments between them. Erase all hidden lines.

35. **Multiple Choice** Identify the dilation shown as an enlargement or reduction and find its scale factor.

- **A** enlargement; \( k = 2 \)
- **B** enlargement; \( k = \frac{1}{3} \)
- **C** reduction; \( k = \frac{1}{3} \)
- **D** reduction; \( k = \frac{1}{2} \)
- **E** reduction; \( k = 3 \)

36. **Multiple Choice** In the diagram shown, the center of the dilation of \( \square JKLM \) is point \( C \). The length of a side of \( \square J'K'L'M' \) is what percent of the length of the corresponding side of \( \square JKLM \)?

- **A** 3%  
- **B** 12%  
- **C** 20%  
- **D** 33\( \frac{1}{3} \)%  
- **E** 300%

37. **Challenge** A polygon is reduced by a dilation with center \( C \) and scale factor \( \frac{1}{k} \). The image is then enlarged by a dilation with center \( C \) and scale factor \( k \). Describe the size and shape of this new image.
**Mixed Review**

**Using the Pythagorean Theorem** Refer to the triangle shown to find the length of the missing side by using the Pythagorean Theorem. (Review 1.3 for 9.1)

38. \(a = 5, b = 12\)  
39. \(a = 8, c = 2\sqrt{65}\)  
40. \(b = 2, c = 5\sqrt{5}\)  
41. \(b = 1, c = \sqrt{50}\)  
42. Find the geometric mean of 11 and 44. (Review 8.2 for 9.1)

**Determining Similarity** Determine whether the triangles can be proved similar or not. Explain your reasoning. (Review 8.4 and 8.5)

43. \(\triangle ABC\)  
44. \(\triangle DEF\)

**Quiz 3**

Use the figure to complete the proportion. (Lesson 8.6)

1. \(\frac{AC}{CE} = \frac{AB}{?}\)  
2. \(\frac{BD}{BF} = \frac{?}{CG}\)  
3. \(\frac{EG}{AG} = \frac{DF}{?}\)  
4. \(\frac{GA}{EA} = \frac{?}{DA}\)

In Exercises 5 and 6, identify the dilation and find its scale factor. (Lesson 8.7)

5.  
6. \(\triangle JKL\) is mapped onto \(\triangle J'K'L'\) by a dilation, with center \(C\). If \(\frac{CJ}{CJ'} = \frac{5}{6}\), then the dilation is (a reduction, an enlargement) and \(\triangle JKL\) is (larger, smaller) than \(\triangle J'K'L'\). (Lesson 8.7)

8. **Enlarging Photos** An 8 inch by 10 inch photo is enlarged to produce an 18 inch by 22\(\frac{1}{2}\) inch photo. What is the scale factor? (Lesson 8.7)
**Chapter Summary**

**WHAT did you learn?**

Write and simplify the ratio of two numbers. (8.1)  
Use proportions to solve problems. (8.1)  
Understand properties of proportions. (8.2)  
Identify similar polygons and use properties of similar polygons. (8.3)  
Prove that two triangles are similar using the definition of similar triangles and the AA Similarity Postulate. (8.4)  
Prove that two triangles are similar using the SSS Similarity Theorem and the SAS Similarity Theorem. (8.5)  
Use proportionality theorems to solve problems. (8.6)  
Identify and draw dilations and use properties of dilations. (8.7)

**WHY did you learn it?**

Find the ratio of the track team’s wins to losses. (p. 461)  
Use measurements of a baseball bat sculpture to find the dimensions of Babe Ruth’s bat. (p. 463)  
Determine the width of the actual Titanic ship from the dimensions of a scale model. (p. 467)  
Determine whether two television screens are similar. (p. 477)  
Use similar triangles to determine the altitude of an aerial photography blimp. (p. 482)  
Use similar triangles to estimate the height of the Unisphere. (p. 494)  
Explain why the diagonal cuts on insulation strips have the same length. (p. 501)  
Understand how the shadows in a shadow puppet show change size. (p. 508)

**How does Chapter 8 fit into the BIGGER PICTURE of geometry?**

In this chapter, you learned that if two polygons are similar, then the lengths of their corresponding sides are proportional. You also studied several connections among real-life situations, geometry, and algebra. For instance, solving a problem that involves similar polygons (geometry) often requires the use of a proportion (algebra). In later chapters, remember that the measures of corresponding angles of similar polygons are equal, but the lengths of corresponding sides of similar polygons are proportional.

**STUDY STRATEGY**

**How did you use your list of real-world examples?**

The list of the main topics of the chapter with corresponding real-world examples that you made following the Study Strategy on page 456, may resemble this one.

**Real-World Examples**

*Lesson 8.1*

**Topic** writing ratios: to find the ratio of wins to losses of the track team.

**Topic** solving proportions: to estimate the weight of a person on Mars.
### VOCABULARY
- ratio, p. 457
- proportion, p. 459
- extremes, p. 459
- means, p. 459
- geometric mean, p. 466
- similar polygons, p. 473
- scale factor, p. 474
- dilation, p. 506
- reduction, p. 506
- enlargement, p. 506

### 8.1 RATIO AND PROPORTION

**EXAMPLE** You can solve a proportion by finding the value of the variable.

\[
\frac{x}{12} = \frac{x + 6}{30}
\]

Write original proportion.

\[
30x = 12(x + 6)
\]

Cross product property

\[
30x = 12x + 72
\]

Distributive property

\[
18x = 72
\]

Subtract 12x from each side.

\[
x = 4
\]

Divide each side by 18.

Solve the proportion.

1. \( \frac{3}{x} = \frac{2}{7} \)
2. \( \frac{a + 1}{5} = \frac{2a}{9} \)
3. \( \frac{2}{x + 1} = \frac{4}{x + 6} \)
4. \( \frac{d - 4}{d} = \frac{3}{7} \)

### 8.2 PROBLEM SOLVING IN GEOMETRY WITH PROPORTIONS

**EXAMPLE** In 1997, the ratio of the population of South Carolina to the population of Wyoming was 47:6. The population of South Carolina was about 3,760,000. You can find the population of Wyoming by solving a proportion.

\[
\frac{47}{6} = \frac{3,760,000}{x}
\]

\[
47x = 22,560,000
\]

\[
x = 480,000 \quad \text{The population of Wyoming was about 480,000.}
\]

5. You buy a 13 inch scale model of the sculpture *The Dancer* by Edgar Degas. The ratio of the height of the scale model to the height of the sculpture is 1:3. Find the height of the sculpture.

6. The ratio of the birth weight to the adult weight of a male black bear is 3:1000. The average birth weight is 12 ounces. Find the average adult weight in pounds.
**8.3  SIMILAR POLYGONS**

**EXAMPLE** The two parallelograms shown are similar because their corresponding angles are congruent and the lengths of their corresponding sides are proportional.

\[ \frac{WX}{PQ} = \frac{ZY}{SR} = \frac{XY}{QR} = \frac{WZ}{PS} = \frac{3}{4} \]

\[ \angle P = \angle R = \angle W = \angle Y = 110^\circ \]

\[ \angle Q = \angle S = \angle X = \angle Z = 70^\circ \]

The scale factor of \( \triangle WXYZ \) to \( \triangle PQRS \) is \( \frac{3}{4} \).

In Exercises 7–9, \( \triangle DEFG \sim \triangle HJKL \).

7. Find the scale factor of \( \triangle DEFG \) to \( \triangle HJKL \).

8. Find the length of \( DE \) and the measure of \( \angle F \).

9. Find the ratio of the perimeter of \( \triangle HJKL \) to the perimeter of \( \triangle DEFG \).

**8.4  SIMILAR TRIANGLES**

**EXAMPLE** Because two angles of \( \triangle ABC \) are congruent to two angles of \( \triangle DEF \), \( \triangle ABC \sim \triangle DEF \) by the Angle-Angle (AA) Similarity Postulate.

Determine whether the triangles can be proved similar or not. Explain why or why not. If they are similar, write a similarity statement.

10.

11.

12.

**8.5  PROVING TRIANGLES ARE SIMILAR**

**EXAMPLES** Three sides of \( \triangle JKL \) are proportional to three sides of \( \triangle MNP \), so \( \triangle JKL \sim \triangle MNP \) by the Side-Side-Side (SSS) Similarity Theorem.
Two sides of $\triangle XYZ$ are proportional to two sides of $\triangle WXY$, and the included angles are congruent. By the Side-Angle-Side (SAS) Similarity Theorem, $\triangle XYZ \sim \triangle WXY$.

Are the triangles similar? If so, state the similarity and a postulate or theorem that can be used to prove that the triangles are similar.

13. 

14. 

8.6 PROPORTIONS AND SIMILAR TRIANGLES

Examples on pp. 498–501

Find the value of the variable.

15. 

16. 

17. 

8.7 DILATIONS

Example: The blue triangle is mapped onto the red triangle by a dilation with center $C$. The scale factor is $\frac{1}{5}$, so the dilation is a reduction.

18. Identify the dilation, find its scale factor, and find the value of the variable.
Chapter Test

In Exercises 1–3, solve the proportion.

1. \( \frac{x}{3} = \frac{12}{9} \)
2. \( \frac{18}{y} = \frac{15}{20} \)
3. \( \frac{11}{10} = \frac{z}{10} \)

Complete the sentence.

4. If \( \frac{5}{2} = \frac{a}{b} \), then \( \frac{5}{a} = \frac{?}{b} \).
5. If \( \frac{8}{x} = \frac{3}{y} \), then \( \frac{8 + x}{x} = \frac{?}{y} \).

In Exercises 6–8, use the figure shown.

6. Find the length of \( EF \).
7. Find the length of \( FG \).
8. Is quadrilateral \( FECD \) similar to quadrilateral \( GFBA \)? If so, what is the scale factor?

In Exercises 9–12, use the figure shown.

9. Prove that \( \triangle RSQ \sim \triangle RQT \).
10. What is the scale factor of \( \triangle RSQ \) to \( \triangle RQT \)?
11. Is \( \triangle RSQ \) similar to \( \triangle QST \)? Explain.
12. Find the length of \( QS \).

In Exercises 13–15, use the figure shown to decide if you are given enough information to conclude that \( JK \parallel LM \). If so, state the reason.

13. \( \frac{LI}{JH} = \frac{MK}{KH} \)
14. \( \angle HJK \equiv \angle HLM \)
15. \( \frac{LH}{JH} = \frac{MH}{KH} \)

16. The triangle \( \triangle RST \) is mapped onto \( \triangle R'S'T' \) by a dilation with \( RS = 24 \), \( ST = 12 \), \( RT = 20 \), and \( R'S' = 6 \). Find the scale factor \( k \), and side lengths \( S'T' \) and \( R'T' \).

17. Two sides of a triangle have lengths of 14 inches and 18 inches. The measure of the angle included by the sides is 45°. Two sides of a second triangle have lengths of 7 inches and 8 inches. The measure of the angle included by the sides is 45°. Are the two triangles similar? Explain.

18. You shine a flashlight on a book that is 9 inches tall and 6 inches wide. It makes a shadow on the wall that is 3 feet tall and 2 feet wide. What is the scale factor of the book to its shadow?
CHAPTER 8

Algebra Review

**EXAMPLE 1**  Simplifying Radicals

Simplify the expression $\sqrt{20}$.

$$\sqrt{20} = \sqrt{4 \cdot 5}$$

Use product property.

$$= 2\sqrt{5}$$

Simplify.

**EXERCISES**

Simplify the expression.

1. $\sqrt{121}$
2. $\sqrt{52}$
3. $\sqrt{45}$
4. $\sqrt{72}$
5. $\sqrt{40}$
6. $\sqrt{27}$
7. $\sqrt{80}$
8. $\sqrt{50}$
9. $\sqrt{243}$
10. $\sqrt{288}$
11. $\sqrt{320}$
12. $\sqrt{225}$

**EXAMPLE 2**  Simplifying Radical Expressions

Simplify the radical expression.

a. $5\sqrt{3} - \sqrt{3} - \sqrt{2}$

$$= 4\sqrt{3} - \sqrt{2}$$

b. $(2\sqrt{2})(5\sqrt{3})$

$$= 2 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{3}$$

$$= 10\sqrt{6}$$

c. $(5\sqrt{7})^2$

$$= 5^2 \cdot \sqrt{7^2}$$

$$= 25 \cdot 7$$

$$= 175$$

**EXERCISES**

Simplify the radical expression.

13. $\sqrt{75} + \sqrt{3}$
14. $\sqrt{50} - \sqrt{18}$
15. $\sqrt{64} - \sqrt{28}$
16. $\sqrt{44} + 2\sqrt{11}$
17. $\sqrt{125} - \sqrt{80}$
18. $\sqrt{242} + \sqrt{200}$
19. $-\sqrt{147} - \sqrt{243}$
20. $\sqrt{28} + \sqrt{63}$
21. $\sqrt{20} + \sqrt{45} - \sqrt{5}$
22. $(\sqrt{13})(\sqrt{26})$
23. $(3\sqrt{14})(\sqrt{35})$
24. $(\sqrt{363})(\sqrt{300})$
25. $(6\sqrt{2})(2\sqrt{2})$
26. $(\sqrt{18})(\sqrt{72})$
27. $(\sqrt{21})(\sqrt{24})$
28. $(\sqrt{32})(\sqrt{2})$
29. $(\sqrt{98})(\sqrt{128})$
30. $(5\sqrt{4})(2\sqrt{4})$
31. $(6\sqrt{5})^2$
32. $(4\sqrt{2})^2$
33. $(8\sqrt{3})^2$
34. $(2\sqrt{3})^2$
35. $(5\sqrt{5})^2$
36. $(10\sqrt{11})^2$
EXAMPLE 3  Simplifying Quotients with Radicals

Simplify the quotient \( \frac{6}{\sqrt{5}} \).

\[
\frac{6}{\sqrt{5}} = \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}
\]

Multiply numerator and denominator by \( \frac{\sqrt{5}}{\sqrt{5}} \), to eliminate a radical in the denominator.

\[
= \frac{6\sqrt{5}}{\sqrt{5} \sqrt{5}}
= \frac{6\sqrt{5}}{5}
\]

EXERCISES

Simplify the quotient.

37. \( \frac{4}{\sqrt{3}} \)  
38. \( \frac{5}{\sqrt{7}} \)  
39. \( \frac{2\sqrt{3}}{\sqrt{6}} \)  
40. \( \frac{2\sqrt{3}}{\sqrt{5}} \)

41. \( \frac{\sqrt{18}}{3\sqrt{2}} \)  
42. \( \frac{4}{\sqrt{8}} \)  
43. \( \frac{16}{\sqrt{24}} \)  
44. \( \frac{\sqrt{5}}{\sqrt{10}} \)

45. \( \frac{4}{\sqrt{12}} \)  
46. \( \frac{3\sqrt{5}}{\sqrt{20}} \)  
47. \( \frac{9}{\sqrt{52}} \)  
48. \( \frac{\sqrt{12}}{\sqrt{24}} \)

49. \( \frac{\sqrt{18}}{\sqrt{10}} \)  
50. \( \frac{\sqrt{32}}{\sqrt{5}} \)  
51. \( \frac{\sqrt{27}}{\sqrt{45}} \)  
52. \( \frac{\sqrt{50}}{\sqrt{75}} \)

EXAMPLE 4  Solving Quadratic Equations

Solve.

\[ x^2 - 5 = 16 \]
\[ x^2 = 21 \]
\[ x = \pm \sqrt{21} \]

Add 5 to each side. Find square roots.

EXERCISES

Solve.

53. \( x^2 = 9 \)  
54. \( x^2 = 625 \)  
55. \( x^2 = 289 \)

56. \( x^2 + 3 = 13 \)  
57. \( x^2 - 4 = 12 \)  
58. \( x^2 - 7 = 6 \)

59. \( 7x^2 = 252 \)  
60. \( 3x^2 = 192 \)  
61. \( 6x^2 = 294 \)

62. \( 4x^2 + 5 = 45 \)  
63. \( 2x^2 + 5 = 23 \)  
64. \( 9x^2 + 7 = 52 \)

65. \( 11x^2 + 4 = 48 \)  
66. \( 6x^2 - 3 = 9 \)  
67. \( 10x^2 - 16 = -6 \)

68. \( 5x^2 - 6 = 29 \)  
69. \( 8x^2 - 12 = 36 \)  
70. \( 5x^2 - 61 = 64 \)

71. \( x^2 + 3^2 = 5^2 \)  
72. \( 7^2 + x^2 = 25^2 \)  
73. \( 5^2 + 12^2 = x^2 \)